C 6159 (Pages : 4) Name.....

Reg. No.....

SECOND SEMESTER B.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION MAY 2016

(UG—CCSS)

Complementary Course

MM 2C 02—MATHEMATICS

Time : Three Hours Maximum : 30 Weightage

Unit I

Answer **all** *questions.*

1. $\frac{d}{dx}(\operatorname{csc} h^{-1} x)$ is equal to :

   (a) $\frac{1}{|x|\sqrt{x^2+1}}$. (b) $\frac{-1}{|x|\sqrt{x^2+1}}$.

   (c) $\frac{1}{x\sqrt{1-x^2}}$. (d) $\frac{-1}{x\sqrt{x^2-1}}$.

2. Write $\sinh^{-1} x$ as a logarithmic function.

3. $\int \cot h\ 5x\ dx$ is equal to :

   (a) $\log \sin h\ 5x + c$. (b) $\log \cosh 5x + c$.

   (c) $\frac{1}{5}\log \sin h\ 5x + c$. (d) $\frac{-1}{5}\log \cosh 5x + c$.

4. Write the n^{th} term of the sequence $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \frac{1}{11}$ .....

5. Find $\lim_{x \to \infty}\left(\frac{\ln x}{x}\right)$.

6. Find the sum of the series $\sum_{n=1}^{\infty} \frac{5(-1)^n}{4^n}$.

7. Test the convergence of the series $\sum_{n=1}^{\infty}\left(\frac{-n}{2n+5}\right)$.

Turn over

8. Define the conditional convergence of the series Σa_n.

9. Find the spherical co-ordinate equation of the sphere $x^2 + y^2 + (z - 1)^2 = 1$.

10. Find the domain and range of the function $w = \sin xy$.

11. Find $\lim_{(x, y) \to (0, 1)} \left(\frac{x - xy + 3}{x^2 y + 5xy - y^3} \right)$.

12. Find $\frac{\partial^2 w}{\partial x \, \partial y}$ if $w = xy + \frac{e^4}{y^2 + 1.}$.

(12 × ¼ = 3 weightage)

Unit II

Answer any **nine** *questions.*

13. Differentiate $t^2 \tan h \frac{1}{t}$ w.r. to t.

14. Find $\int \csc h^2 (5 - x) \, dx$.

15. Find $\int_{-\infty}^{\infty} \frac{2x}{(x^2 + 1)^2} \, dx$.

16. Does $\sum_{n=1}^{\infty} \frac{\log n}{n^{3/2}}$ converge ? Why ?

17. For what value of x the power series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!}$ converges ?

18. Find the Taylor series expansion of $f(x) = x^2 - 2x + 4$ at $x = 2$.

19. Find the polar equation of the circle $x^2 + (y - 3)^2 = 9$.

20. Find the centre and radius of the circle $r = 4 \sin \theta$.

21. Find $\lim_{(x, y) \to (0, 0)} \left(\frac{x^2 - xy}{\sqrt{x} - \sqrt{y}} \right)$.

22. Find f_x, f_y, f_z if $f(x, y, z) = 1 + xy^2 - 2z^2$.

23. Define the linearization of $f(x, y)$ at (x_0, y_0).

24. Evaluate $\int_2^\infty \frac{x + 3}{(x - 1)(x^2 + 1)} dx$.

(9 × 1 = 9 weightage)

Unit III

Answer any **five** *questions.*

25. Evaluate $\int_0^1 \frac{2\,dx}{\sqrt{3 + 4x^2}}$.

26. Show that the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges for $p > 1$ and diverges for $p \leq 1$.

27. Using geometric series prove that $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots\ldots$

28. Find the Maclaurin's series for $\cos x$.

29. Find the points of intersection of the pair of curves $r = 4 \cos \theta$, $r = 1 - \cos \theta$.

30. Find the tangent plane to the surface $z = x \cos y - y e^x$ at (0, 0, 0).

31. Find the direction in which $f(x, y) = \frac{x^2}{2} + \frac{y^2}{2}$ increases most rapidly at (1, 1).

32. Find the area of the region enclosed by $r = 2(1 + \cos \theta)$.

(5 × 2 = 10 weightage)

Unit IV

Answer any **two** *questions.*

33. (a) State and prove the Leibnitz theorem for the alternating series $u_1 - u_2 + u_3 - u_3 + ...$

(b) Investigate the convergence of $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$.

34. Find the points of intersection of the curves $r^2 = 4 \cos \theta$ and $r = 1 - \cos \theta$.

35. Find the region enclosed by the cardioid $r = 2 \cos \theta + 1$.

(2 × 4 = 8 weightage)