Name...

Reg. No.......................................

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2018

(CUCBCSS–UG)

Complementary Course

MAT 2C 02—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

Part A (Objective Type)

Answer **all twelve** *questions.*
Each question carries 1 mark.

1. Write an example for a sequence which has no upper bound.

2. Find the domain of the function $w = xy \ln z$.

3. Define the level surface of a function f.

4. State two path test for non-existence of limit.

5. If $\sum\limits_{n=1}^{\infty} |a_n|$ converges then $\sum\limits_{n=1}^{\infty} a_n$.

6. $\dfrac{d}{dx} \sinh x = $ ———.

7. Write $\tanh x$ in terms of exponential function.

8. Find $\lim\limits_{n \to \infty} \sqrt[n]{n}$.

9. $\int \cosh 2x = $ ———.

10. Find $\lim\limits_{(x,y) \to (1,1)} \dfrac{x^2 + 2y}{3x - 2}$.

11. Find $\dfrac{\partial}{\partial x} \sin 2xy$.

12. Define conditional convergence of a series.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 marks.

13. Investigate the convergence of $\int_0^{\infty} e^{-x^2} dx$.

14. Show that $\lim\limits_{n \to \infty} k = k$, where k is a constant.

Turn over

15. Find $\lim\limits_{n \to \infty} \dfrac{\cos n}{n}$.

16. Find $\lim\limits_{(x,y) \to (0,0)} \dfrac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$.

17. Show that the function $f(x, y) = \dfrac{2x^2 y}{x^4 + y^2}$ has no limit as (x, y) approaches $(0, 0)$.

18. Find $\dfrac{\partial f}{\partial y}$ if $f(x, y) = y \sin xy$.

19. Use chain rule to find the derivative of $w = xy$ with respect to t along the path $x = \cos t, y = \sin t$. What is the derivative's value at $t = \pi/2$?

20. Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$.

21. Show that if u is a differentiable function of x whose values are greater than 1, then
$$\frac{d}{dx}\left(\cosh^{-1} u\right) = \frac{1}{\sqrt{u^2 - 1}} \frac{du}{dx}.$$

22. Graph the sets of points whose co-ordinates satisfies the condition $2\pi/3 \le \theta \le 5\pi/6$ (no restriction on r).

23. Find a polar equation for the circle $x^2 + (y - 3)^2 = 9$.

24. Find the directrix of the parabola $r = \dfrac{25}{10 + 10 \cos\theta}$.

$(9 \times 2 = 18 \text{ marks})$

Part C (Short Essay Type)

*Answer any **six** questions.*
Each question carries 5 marks.

25. Compare $\int_1^\infty \dfrac{dx}{x^2}$ and $\int_1^\infty \dfrac{dx}{1 + x^2}$ with limit comparison test.

26. Determine whether the series $\sum\limits_{n=1}^{\infty} \dfrac{1}{n^2}$ convergent or divergent.

27. Find the linearization of the function $f(x, y) = x^2 + y^2 + 1$ at $(0, 0)$.

28. Express $\dfrac{\partial w}{\partial r}$ and $\dfrac{\partial w}{\partial s}$ in terms of r and s if $w = x^2 + y^2, x = r - s$ and $y = r + s$.

29. Find the area of the region in the plane enclosed by the cardioid $r = 2 (1 + \cos\theta)$.

30. Show that $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} + \dfrac{\partial^2 f}{\partial z^2} = 0$ if $f(x, y, z) = e^{3x+4y} \cos 5z$.

31. Find the Maclaurin series for the function $f(x) = xe^x$.

32. Does series $\sum\limits_{n=1}^{\infty} \dfrac{\ln n}{n^{3/2}}$ convergent.

33. Find the surface area generated by revolving the curves $x = t + \sqrt{2}$, $y = \dfrac{t^2}{2} + \sqrt{2}t$, $-\sqrt{2} \le t \le \sqrt{2}$ about y-axis.

(6 × 5 = 30 marks)

Part D (Essay Type)

*Answer any **two** questions.*
Each question carries 10 marks.

34. Find the length of the curve $y = \dfrac{1}{3}(x^2 + 2)^{3/2}$ from $x = 0$ to $x = 3$.

35. Find the points of intersection of $r^2 = 4 \cos \theta$ and $r = 1 - \cos \theta$.

36. Find the critical points of $f(x) = x^{1/3}(x - 4)$. Identify the intervals on which f is increasing and decreasing. Find the functions's local and absolute extrema values.

(2 × 10 = 20 marks)