C 62623 (Pages : 3) Name.................................

Reg. No.................................

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2019

(CUCBCSS–UG)

Mathematics

MAT 2C 02—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Types)

Answer **all twelve** *questions.*

1. Define a sequence.

2. Fill in the blanks : $\frac{d}{dx}\cosh^3(3x) =$ ————————.

3. For what values of real numbers x, does the series $\sum_{n=1}^{\infty} \sin^n x$ converge ?

4. Fill in the blanks : The polar equation of the circle with centre origin and radius a is ————————.

5. Find the n^{th} term of the sequence 2, – 2, 2, – 2 ————————.

6. Fill in the blanks : If $f(x,y) = 1 - \sinh(1 - xy)$, then $f_x(1,1) =$ ————————.

7. Fill in the blanks : If f is continuous on $[a,b)$, then $\lim_{c \to b} \int_a^c f(t)\,dt =$ ————————.

8. Write explicitly the ratio test for the convergence of the series $\sum_{n=0}^{\infty} a_n$.

9. State alternating series test of Leibniz.

10. Define $\frac{\partial}{\partial x} f(x,y)$ using limit.

11. The power series $\sum_{n=0}^{\infty} a_n (x-a)^n$ always converges to a_0 when $x =$ ————————.

12. What do you mean by linearization of a function in two variables at a point.

(12 × 1 = 12 marks)

Part B (Short Answer Types)

Answer any **nine** *questions.*

13. Evaluate $\int_0^1 \sinh^2 x \, dx$.

14. Test the convergence of the integral $\int_0^{1/2} \frac{1}{1-2x} dx$.

15. State the non-decreasing sequence theorem.

16. Describe the level surface of the function $f(x,y,z) = \sqrt{x^2+y^2+z^2-1}$.

17. Graph the sets of points whose polar co-ordinates satisfy the condition $0 \le r \le 2$.

18. Evaluate $\int_0^1 \frac{3dx}{\sqrt{4+9x^2}}$.

19. Find $\tanh x$, if $\cosh x = \frac{17}{15}$, $x > 0$.

20. Show that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$ if $f(x,y) = \log\sqrt{x^2+y^2}$.

21. Find a cylindrical co-ordinate equation for the surface $x^2 + (y-3)^2 = 9$.

22. Find $\frac{\partial z}{\partial r}$ if $z = x + 2y$, $x = \frac{r}{s}$ and $y = 2rs$.

23. Find $\lim_{n \to \infty} \frac{n}{2n+1}$.

24. Write the Maclaurin series for $\sin x$.

(9 × 2 = 18 marks)

Part C (Short Essay Types)

Answer any **six** *questions.*

25. Find the length of the curve $y = \frac{2\sqrt{2}}{3} x^{\frac{3}{2}} - 1$ from $x = 0$ to $x = 1$.

26. Find the limit of the function $f(x,y) = \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$ as (x,y) tends to (0, 0).

27. Replace the polar equation $r = \frac{4}{2\cos\theta - \sin\theta}$ by equivalent Cartesian equation and the draw the graph in Cartesian form.

28. Find a power series for log $(1 + x)$ and find the radius of convergence of that series.

29. Show that $\tanh^{-1} x = \frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$.

30. Find the volume of the solid of revolution when the region between the parabola $x = y^2 + 1$ and the line $x = 3$ is revolved about the line $x = 3$.

31. Find the sum of the series $\sum_{n=1}^{\infty} \frac{2^n - 1}{4^n}$.

32. Find the radius and interval of convergence of the series : $\sum_{n=0}^{\infty} (-1)^n (2x-1)^n$.

33. Evaluate : $\int \frac{\cosh^4 \sqrt{x}}{\sqrt{x}}\, dx$.

(6 × 5 = 30 marks)

Part D (Essay Types)

Answer any **two** *questions.*

34. Show that the function $f(x,y) = \frac{2xy}{x^2 + y^2}$ when $(x,y) \neq (0,0)$ and 0, otherwise is continuous everywhere except at the origin.

35. (a) Find the linearization of the function $f(x,y) = x^2 - xy + y^2/2 + 3$ at (3, 2).

(b) Find the area of the region enclosed by the cardioid : $r = 2(1 + \cos\theta)$.

36. Find the area of the surface generated by revolving the curve $y = x^3/9, 0 \le x \le 2$ about the x-axis.

(2 × 10 = 20 marks)