D 3051 (Pages : 3) Name.....

Reg. No.....

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2009

Mathematics—Complementary Course

MM IC 01—MATHEMATICS

(C.S.S.)

Time : Three Hours Maximum Weightage : 30

I. Objective Type Questions - Answer all 12 questions (12 × ¼ = 3 weightage)

1. $P(x)$ and $Q(x)$ are polynomials and $Q(c) \neq 0$ then $\lim_{x\to c}\frac{P(x)}{Q(x)}$ = ----
2. 3. $\lim_{x\to 2}\frac{f(x)-5}{x-2} = 4$ then $\lim_{x\to 2} f(x) = \cdots$
3. The function $f(x) = \dfrac{\cos x}{x}$ is not continuous at $x =$ ---
4. The slope of the curve $y = \dfrac{1}{x}$ at $x = 1$ equals ...
5. If $\lim\limits_{x\to 0} f(x) = \dfrac{1}{2}$, then $\lim\limits_{x\to 0}\dfrac{f(x)\cos x}{x-1}$ = ...
6. The absolute maximum value of $f(x) = -x-4$, $-4 \le x \le 1$, is at $x =$ .....
7. If f '(x) > 0 for every x in (a, b), then f is ..... in (a, b).
8. The horizontal asymptote of the curve $y = \dfrac{1}{x}$ is ...
9. If f is continuous and $\int_1^2 f(x)dx$ = - 4 and $\int_1^5 f(x)dx = 6$ then $\int_2^5 f(x)dx$ = ...
10. If f is integrable on [a, b], then the average value of f on [a, b] is $av(f)$ ....
11. $\frac{d}{dx}\left(\int_0^x \frac{1}{1+t^2dt}\right)$ = ...
12. If f is smooth on $[a,b]$, the length of the curve $y = f(x)$ from a to b is L = ...

II. Short answer type questions – Answer all 9 questions (9 × 1 = 9 weightage)

13. If $f(x) = 2x-4$ and $x_0 = 5$, $\epsilon = 0.2$ and $L = 6$, find $\delta > 0$, such that $0 < |x - x_0| < \delta$ implies $|f(x) - L| < \varepsilon$.

Turn over

14. For what values of a is $f(x)=\begin{cases} x^2-1 & x<3 \\ 2ax & x\geq 3 \end{cases}$ continuous at $x=3$?

15. Find the value of c that satisfies the mean value theorem for the function $f(x)=x^2+2x-1$ on $[0,1]$.

16. If x moves from left to right through the point $c=2$, is the graph of $f(x)=x^3-3x+2$ rising or falling? Justify your answer.

17. Use Sandwich theorem to find the asymptotes of $y=2+\frac{\sin x}{x}$.

18. Find the inflection point of the curve $f(x)=x^3-3x+3$.

19. Consider the function $f(x)=x^2-1$ on $[0,2]$. Partition the interval into four subintervals of equal length. Find the Riemann sum $\sum_{k=1}^{4} f(c_k)\,\Delta c_k$ where c_k is the left end point.

20. State the mean value theorem for definite integrals.

21. Find the area between the curves $y=\sec^2 x$ and $y=\sin x$ from 0 to $\frac{\pi}{4}$.

III. Short essay or paragraph questions – Answer any 5 questions from 7 (5 × 2 = 10 weightage)

22. Draw the graph of the function

$$f(x)=\begin{cases} 3-x, x<2 \\ 2, x=2 \\ \frac{x}{2}, x>2 \end{cases}$$

Find the limits or explain why they do not exist?

a) $\lim_{x\to 2^+} f(x)$ b) $\lim_{x\to 2^-} f(x)$

c) Does $\lim_{x\to 2} f(x)$ exist? If so what is it? If not. why?

23. If a function is differentiable at $x = c$, prove that it is continuous at $x = c$. Is the converse true? Justify your answer.

24. If b, c, d are constants, for what value of b will the curve $y = x^3 + bx^2 + cx + d$ has a point of inflection at $x = 1$.

25. Find the average value of $f(x) = 3x^2 - 3$ on $[0,1]$. At what points in the interval does this function assume its average value?

26. Find the area of the region between the curve $y = 4 - x^2$, $0 \le x \le 3$, and the x-axis.

27. The region between the curve $y = 2\sqrt{x}$, $0 \le x \le 2$, and the x-axis is revolved about the x-axis. Find the volume of the solid generated.

28. Applying L'Hospital's rule find $\lim_{x\to\infty} x^{1/x}$

IV. Essay questions – Answer 2 questions from 3 (2 × 4 = 8 weightage)

29. Sketch the graph of y = sin (1/x) and show that y = sin (1/x) has no limit as x approaches zero from either side.

Or

If f and g are continuous functions, then prove that
a) f + g is continuous b) fg is continuous

30. Plot the graph and find the derivative at each critical point and determine the local extreme values.

$$y = \begin{cases} -x^2 - 2x + 4 \ , x \le 1 \\ -x^2 + 6x - 4 \ , x > 1 \end{cases}$$

31. Use definite integral to estimate the sum of the square roots of the first n positive integers, $\sqrt{1} + \sqrt{2} + \cdots + \sqrt{n}$.

Or

Find the length of the curve $y = x^{3/2}$ from $x = 0$ to $x = 4$.