

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2010

(CCSS)

MM 1C 01—MATHEMATICS

Maximum Weightage : 30

Time : Three Hours

Part A (Objective Type Questions)

Answer all twelve questions.

Each bunch of four questions carries Weight.

1. Evaluate $\lim_{x \rightarrow -5} \frac{y^2}{5-y}$.
2. Find $\lim_{x \rightarrow 1} \frac{1-\sqrt{x}}{1-x}$.
3. Determine $\lim_{x \rightarrow 1} + \sqrt{\frac{x-1}{x+2}}$.
4. Find the point of discontinuity of the function $y = \frac{x+2}{\cos x}$.
(4 × ¼ = 1 weight)
5. Find the slope of the curve $f(x) = \frac{x}{x-2}$ at (3, 3).
6. Determine the point(s) at which the curve $y = x^3 + \frac{4}{3}x^2 - 5x + 1$ has horizontal tangents.
7. Find the derivative of $y = (x^2 + 1)(x^3 + 2)$.
8. State the quotient rule of differentiation.
(4 × ¼ = 1 weight)
9. Find the absolute maximum of $y = x^2$ in [0, 2].
10. Where does the function $y = \sec x$ have vertical asymptotes ?
11. Evaluate $\sum_{k=1}^1 \frac{k-1}{k}$.
12. Find the average value of $f(x) = 4 - x^2$ on [0, 3].
(4 × ¼ = 1 weight)

Turn over

Short Answer Type Questions

Answer all nine questions.

Each question carries 1 weight.

13. Find the average rate of change of the function $f(x) = x^3 + 1$ in the interval $[2, 3]$.
14. If $1 - \frac{x^2}{4} \leq u(x) \leq 1 + \frac{x^2}{2}$ for all $x \neq 0$, find $\lim_{x \rightarrow 0} u(x)$.
15. Show that $\lim_{x \rightarrow a} k = k$.
16. Check the continuity of the function $f(x) = |x|$ in \mathbb{R} .
17. Find the second order derivative of $y = \frac{(x-1)(x^2-2x)}{x^4}$.
18. Check whether the function $f(x) = \sqrt{x(x-1)}$ satisfy the hypotheses of Mean Value Theorem in $[0, 1]$.
19. Find the critical points of $f(x) = x^{1/3}(x-4)$.
20. Determine the interval on which the function $f(x) = x^3 + 12x + 5, x \in [-3, 3]$ is increasing.
21. Find the area of the region between the curve $y = 3x^2$ and the x -axis on the interval $[0, b]$.
($9 \times 1 = 9$ weight)

Short Essay Questions

Answer any five questions.

Each question carries 2 weights.

22. At seconds after lift-off, the height of a rocket is $3t^2$ ft. How fast is the rocket climbing at 10 sec.?
23. Can $\lim_{x \rightarrow c} \frac{f(x)}{g(x)}$ exist even if $\lim_{x \rightarrow c} f(x) = 0$ and $\lim_{x \rightarrow c} g(x) = 0$? Give reasons for your answer.
24. Explain why the function $f(x) = \sin\left(\frac{1}{x}\right)$ has no continuous extension to $x = 0$.
25. Find $\frac{dy}{dx}$, if $y = 2x^3$ using the definition of derivatives.
26. The curves $y = x^2 + ax + b$ and $y = cx - x^2$ have a common tangent line the point $(1, 0)$. Find a and c .
27. Suppose the derivative of the function $y = f(x)$ is $y' = (x-1)^2(x-2)(x-4)$. What points, if any, does the graph of f have a local minimum, local maximum, or point of inflection?

28. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 1$, $x = 4$ about the line $y = 1$.

(5 × 2 = 10 weights)

Essay Questions

*Answer any two questions.
Each question carries 4 weights.*

29. Graph the function

$$f(x) = \begin{cases} \sqrt{1-x^2} & \text{if } 0 \leq x \leq 1 \\ 1 & \text{if } 1 \leq x \leq 2. \\ 2 & \text{if } x = 2 \end{cases}$$

- What are the domain and range of f ?
- At what points c , if any, does $\lim_{x \rightarrow c} f(x)$ exist?
- At what points does only the left-hand limit exist?
- At what points does only the right-hand limit exist?
- At what points does the function is continuous?

30. The first derivative of a continuous function $y = f(x)$ is $y' = (x^2 - 2x)(x - 5)^2$. Find y'' and sketch the general shape of the graph of f .

- What are the critical points of f ?
- On what intervals is f increasing or decreasing?
- At what points, if any, does f assume local maximum and minimum values?
- What are the points of inflexion of f ?

31. Find :

- The length of the curve $y = \left(\frac{x}{2}\right)^{2/3}$ from $x = 0$ to $x = 2$.
- The area of the surface generated by revolving the curve $y = x^3$, $0 \leq x \leq 1/2$, about the x -axis.
- The volume of the solid generated by revolving the region bounded by the curves $x = \sqrt{y}$, $x = -y$, $y = 2$.

(2 × 4 = 8 weights)