22290 (Pages : 4) Name.................................

Reg. No.................................

FIRST SEMESTER B.Sc. DEGREE EXAMINATION JANUARY 2012

(CCSS)

Statistics—Complementary

ST 1C 01—PROBABILITY THEORY

Three Hours Maximum : 30 Weightage

Part A

Answer all *questions.*
A bunch of 4 *questions carries weightage* 1.

1. If A and B are independent events then which of the following is not true ?

   (a) $P(AB) = P(A) \cdot P(B)$ (b) $P(A^c B) = P(A^c) \cdot P(B)$

   (c) $P(A \cup B) = P(A) + P(B)$ (d) $P(A^c B^c) = P(A^c) \cdot P(B^c)$

2. A coin is tossed 5 times and observed the sides coming up. The number of sample points in the sample space is :

   (a) 5. (b) 2.

   (c) 8. (d) 32.

3. $P\left(A^c|B\right)$ is :

   (a) $1-(P(A/B)$. (b) $1-P(B/A)$.

   (c) $1-P(B^c/A)$. (d) $P(A^C/B^C)$.

4. If A and B are two events, then $(A \cap B)^c$ is :

   (a) $A^c \cap B^c$ (b) $A^c \cup B^c$.

   (c) $A \cup B$ (d) $A \cap B$.

5. An unbiased die is thrown. Probability of getting a number greater than 4 is :

   (a) $\frac{1}{2}$ (b) $1/6$

   (c) $4/6$ (d) $1/3$

Turn over

6. If $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A/B) = \frac{1}{3}$ then P (AB) is :

(a) $\frac{1}{2}$.

(b) $\frac{1}{6}$.

(c) $\frac{1}{4}$.

(d) $\frac{1}{3}$.

7. Consider the following p.m.f.

x	-1	0	2
$f(x)$	k	$2k$	$3k$

Then the value of k is :

(a) 1

(b) $\frac{1}{2}$.

(c) $\frac{1}{6}$.

(d) $\frac{1}{3}$

8. If X is a continuous random variable, then $Y = 2X + 3$ is a

(a) Discrete random variable.

(b) Continuous random variable.

(c) Constant.

(d) None of these.

9. If X is a random variable, then $P(X \leq x)$ is known as :

(a) df.

(b) pmf.

(c) pdf.

(d) MGF.

10. E (X) exists if :

(a) $\int_{-\infty}^{\infty} x\, f(x)\, dx < \infty$

(b) $E(|X|) < \infty$.

(c) Characteristic function exists.

(d) None of these.

11. $E\left(e^{itX}\right)$ is known as :

(a) MGF.

(b) Characteristics function.

(c) df.

(d) pdf.

12. $E(X^r)$ is known as :

(a) Variance. (b) Central moment.

(c) Skewness. (d) Raw moment.

(12 × ¼ = 3 weightage)

Part B

Answer **all** *questions.*
Each question carries weightage 1.

13. Give an example of two mutually exclusive events.
14. Give the classical definition of probability.
15. State multiplication theorem of probability.
16. Two unbiased dice are thrown. Find the probability that the product of the numbers coming up is 12.
17. Define probability mass function of a random variable.
18. Show that $E(X^2) \geq [E(X)]^2$.
19. Define MGF of a random variable.
20. Show that $E(XY) = E(X) \cdot E(Y)$, when X and Y are independent random variables.
21. Find the pmf of $Y = 2X + 5$, when the pmf of X is given by :

x	0	1	2
$f(x)$	1/3	1/3	1/3

(9 × 1 = 9 weightage)

Part C

Answer any **five** *questions.*
Each question carries weightage 2.

22. State and prove addition theorem of probability.
23. If P (A) = 0.4, P (B) = 0.3 and A (AB) = 0.2 find

    (a) P (A/B) (b) $P(A^c / B^c)$.

24. A continuous random variable X has the following pdf :

$$f(x) = \begin{cases} A & , -1 < x < 1 \\ 0 & , \text{elsewhere} \end{cases}$$

    (a) Find A

    (b) Find $P(X = 0)$ and $P(X \geq 0)$

25. Establish the relationship between raw moments and central moments of a random variable.

26. Find the MGF of a random variable with the following pdf $f(x) = \frac{1}{2} e^{-|x|}, -\infty < x < \infty$

27. If X is a random variable with the following pmf :

x	-2	0	2
$f(x)$	1/4	1/2	1/4

Obtain β_1 and β_2, the measures of skewness and kurtosis.

28. Let X be a random variable with following pdf :

$$f(x) = \begin{cases} ke^{-2x} & , 0 < x ,< \infty \\ 0 & , \text{elsewhere} \end{cases}$$

(a) Find K

(b) Obtain the pdf of $Y = X^2$.

(5 × 2 = 10 w

Part D

Answer any **two** *questions.*
Each question carries weightage 4.

29. (a) State and prove Baye's theorem.

(b) If P (A) = 0.4, P (B) = 0.3, P (AB) = 0.2 find the probability of :

(i) at least one of the events occurs.

(ii) exactly one of the events occurs.

30. Let X be a random variable with pmf

$$f(x) = \begin{cases} k \frac{\theta^x}{x!} & ; x = 0, 1, \ldots \text{and } \theta > 0 \\ 0 & ; \text{elsewhere} \end{cases}$$

(a) Find K

(b) Find MGF of X.

(c) Show that E (X) = var (X) = θ

31. The following is the pdf of a continuous random variable.

$$f(x) = \begin{cases} k(1-x^2), & -1 < x < 1 \\ 0 & ; \text{elsewhere} \end{cases}$$

(a) Find K

(b) Obtain mean and variance of K.

(c) Find the pdf of Y = 2X.

(2 × 4 = 8 we