

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2009

Complementary Course for Statistics

ST IC 01—PROBABILITY THEORY

(C.S.S.)

Time : Three Hours

Maximum Weightage : 30

Part A*Answer all the questions weightage 1 for branch of 4.*

1. Suppose a student is selected at random from 80 students where 30 are taking Mathematics, 20 are taking Chemistry, and 10 are taking Mathematics and Chemistry. The probability that the student is taking Mathematics or Chemistry is :

(a) $\frac{3}{8}$.

(b) $\frac{1}{4}$.

(c) $\frac{1}{8}$.

(d) $\frac{1}{2}$.

2. One card is selected at random from 25 cards numbered 1 to 25, the probability that the number on the card is divisible by 3.

(a) $\frac{12}{25}$.

(b) $\frac{8}{25}$.

(c) $\frac{4}{25}$.

(d) $\frac{16}{25}$.

3. Under which of the following functions does $S = \{a_1, a_2, a_3\}$ become a probability space :

(a) $p(a_1) = 0.3, p(a_2) = 0.4, p(a_3) = 0.5$.

(b) $p(a_1) = 0.3, p(a_2) = 0.2, p(a_3) = 0.5$.

(c) $p(a_1) = 0.7, p(a_2) = -0.2, p(a_3) = 0.5$.

(d) None of these.

4. One bag contains 5 red and 3 white balls. A second contains 4 red and 7 black balls. If one ball is drawn at random from each bag, what is the probability that both are of the same colour ?

(a) $\frac{3}{25}$.

(b) $\frac{5}{22}$.

(c) $\frac{7}{22}$.

(d) $\frac{9}{22}$.

Turn over

5. If $P(A \cap B \cap C) = P(A) P(B) P(C)$, then :

- (a) A, B, C are equally likely events. (b) A, B, C are exhaustive events.
 (c) A, B, C are independent events. (d) None.

6. Evaluate k , if the following function is a probability mass function :

$$f(x) = \begin{cases} k; & x = 1, 2, 3, 4, 5, 6 \\ 0; & \text{otherwise} \end{cases}$$

- (a) 6. (b) $\frac{1}{6}$.
 (c) 3. (d) $\frac{1}{3}$.

7. A random variable X has the following probability function :

x	:	0	1	2	3	4	5	6	7
$f(x)$:	0	$2k$	$3k$	k	$2k$	k^2	$7k^2$	$2k^2 + k$

then $p(0 < X < 5)$.

- (a) $\frac{1}{5}$. (b) $\frac{2}{5}$.
 (c) $\frac{3}{5}$. (d) $\frac{4}{5}$.

8. X is a random variable such that $f(x) = 2x$; $0 < x < 1$ and $f(x) = 0$ otherwise, then $E(X)$ is equal to

- (a) $\frac{1}{3}$. (b) $\frac{2}{3}$.
 (c) 1. (d) $\frac{4}{3}$.

9. If it rains, a taxi driver can earn Rs. 100 per day. If it is fair, he can lose Rs. 10 per day. If the probability of rain is 0.4, what is his expectation ?

- (a) 34. (b) 46.
 (c) -34. (d) -46.

10. Where coefficient of skewness is zero, the distribution is :

- (a) J-shaped, (b) V-shaped.
 (c) Symmetrical. (d) L-shaped.

11. Where $\beta_2 = 3$, the distribution is :
- (a) Leptokurtic. (b) Platykurtic.
 (c) Mesokurtic. (d) None of these.
12. A variable X has the probability density function $f(x) = 6x(1-x)$ for $0 \leq x \leq 1$, then its variance is :
- (a) 0.5. (b) 0.05.
 (c) 0.005. (d) None of these.

(12 × ¼ = 3 weightage)

Part B

Answer all the questions, weightage 1.

13. State the axiomatic definition of probability.
14. For any events A, B, C , what is $P(A \cup B \cup C)$?
15. Two men A and B fire at a target. Suppose $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{5}$ denote their probabilities of hitting the target, then the probability that one of them hits the target.
16. Define conditional probability and independent events.
17. Define probability density function for the continuous random variable.
18. A box contains six tickets. Two of the tickets carry a prize of Rs. 5 each and other four prizes of Re. 1. If one ticket is drawn, what is the expected value of the prize?
19. State the properties of distribution function.
20. Let X have the density function $f(x) = 0.75(1-x^2)$; if $-1 \leq x \leq 1$ and zero otherwise. Find the distribution function. "Kurtosis".
21. Explain the term "Kurtosis".

(9 × 1 = 9 weightage)

Part C

Answer any five questions, weightage 2.

22. Let $S = \{a, b, c, d\}$ be an equiprobable space and consider the events $A = \{a, b\}$, $B = \{b, d\}$, $C = \{c, d\}$. Show that A, B, C are pair wise independent but they are not total independent.
23. State and prove addition and multiplication theorem of probability for two events.
24. Three groups of workers containing 3 men and one woman, 2 men and 2 women and one man and 3 women respectively, one worker is selected at random from each group. What is the probability that the group selected consists of one man and 2 women?

Turn over

25. Let X be the ratio of sales to profits of some firm. Assume that X has the distribution function :

$$F(x) = \begin{cases} 0 & \text{if } x < 2 \\ \frac{(x^2 - 4)}{5} & \text{if } 2 \leq x < 3 \\ 1 & \text{if } x \geq 3 \end{cases}$$

- Find the density. What is the probability that X is between 2.5 (40% profit) and 5 (20% profit) ?
26. Define a random variable. What do you mean by expectation and variance of a random variable?
27. What is Kurtosis? How is it measured? Explain.
28. Explain the addition and multiplication theorem.

(5 × 2 = 10 weightage)

Part D

Answer any **two** questions, weightage 4.

29. State and prove Bayes theorem.
30. Let X be random variable with probability density function :

$$f(x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

Obtain the distribution function, k^{th} order raw moment and moment generating function. What is its expectation?

31. What is symmetric distribution? What is skewness? How do you test the presence of skewness in a distribution? The first four moments of a distribution about the value 20 are $-1, 24, 18, 509$. Find the coefficient of skewness and interpret.

(2 × 4 = 8 weightage)