D 32500 **(Pages : 4)** **Name..**

Reg. No.......................................

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, JANUARY 2013

(CCSS)

Statistics—Complementary Course

ST 1C 01—PROBABILITY THEORY

Time : Three Hours Maximum : 30 Weightage

I. Objective type questions. Answer all *twelve* questions :

1 A single letter is selected at random from the word "PROBABILITY". The probability that it is vowel is :

(a) $\frac{2}{11}$. (b) $\frac{3}{11}$.

(c) $\frac{4}{11}$. (d) None of these.

2 For any two events A and B, the value of $P(A \cap B) + P(\bar{A} \cup \bar{B})$ is :

(a) Equal to one. (b) Less than one.

(c) Greater than one. (d) Greater than or equal to one.

3 If $P(A) = x$, $P(B) = y$ and $P(A \cap B) = z$, then $P(\bar{A} \cup B) =$

(a) $1 - x - y + z$. (b) $1 - x + y$.

(c) $1 - y + z$. (d) $1 - x + z$.

4 If A and B are mutually exclusive events, then $P(A \cup B) =$

(a) P(A). P (B) (b) 0.

(c) P (A) + P (B). (d) P (A). P (B | A).

5 For any two events A and B each with positive probabilities, P (A | B) is :

(a) $< P(A)$. (b) $> P(A)$.

(c) $\leq P(A)$. (d) $\geq P(A)$.

Turn over

6 If X is a continuous random variable, then $P(a < X < b)$:

(a) $< P(a < X \leq b)$. (b) $> P(a < X \leq b)$.

(c) $< P(a \leq X \leq b)$. (d) $= P(a \leq X \leq b)$.

7 For any reals $x < y$, the distribution function F of a random variable X statistics the inequality :

(a) $F(x) < F(y)$. (b) $F(x) > F(y)$.

(c) $F(x) \leq F(y)$. (d) $F(x) \geq F(y)$.

8 If X is a non-negative continuous random variable, the range of E (X) is :

(a) $(-\infty, \infty)$. (b) $(0, 1)$.

(c) $[0, 1]$. (d) $[0, \infty]$.

9 If X is a random variable with mean μ and variance σ^2, then $E(X - b)^2$ is minimum when $b =$.

(a) 0. (b) 1.

(c) μ. (d) σ^2.

10 If the third central moment is positive, then the curve is :

(a) Symmetric. (b) Positively skewed.

(c) Negatively skewed. (d) U-shaped.

11 If X assumes only positive values, then $E\left(X^{1/2}\right)$ is :

(a) $= [E(X)]^{1/2}$. (b) $< [E(X)]^{1/2}$.

(c) $> [E(X)]^{1/2}$. (d) $\leq [E(X)]^{1/2}$.

12 The measure of Kurtosis β_2 is always :

(a) > 1. (b) < 1.

(c) ≥ 1. (d) ≤ 1.

(12 × ¼ = 3 weightage)

II. Short answer type questions. Answer all *nine* questions :

13 Define Sample space.

14 What are the limitations of statistical definition of probability.

15 State addition theorem of probability.

16 Define conditional probability.

17 Define distribution function of a random variable.

18 Define Central moments.

19 What is Kurtosis ?

20 State the necessary and sufficient conditions for a function to be a characteristic function.

21 Distinguish between discrete and continuous type random variables.

(9 × 1 = 9 weightage)

III. Short essay *or* paragraph questions. Answer any *five* questions :

22 Distinguish between equally likely events and independent events.

23 Is pair-wise independence implies mutual independence ? Justify your claim.

24 Examine whether the following is a distribution function :

$$F(x) = \begin{cases} 0 & , \quad x < -2 \\ \frac{1}{2}\left(\frac{x}{2}+1\right), & -2 \le x \le 2 \\ 1 & , \quad x > 2 \end{cases}$$

Also find its probability density function.

25 State and establish the multiplication theorem of expectation.

26 If X_1 and X_2 are independent random variables, show that $\text{Var}(X_1 + X_2) = \text{Var}(X_1 - X_2)$.

27 Obtain the relationship between raw moments and central moments.

28 State and establish any two properties of characteristic function.

(5 × 2 = 10 weightage)

IV. Essay questions. Answer any *two* questions :

29 Out of 1000 persons born only 900 reach the age of 15, and out of every 1,000 who reach the age of 15, 950 reach the age of 50. Out of every 1,000 who reach the age of 50, forty die in one year. What is the probability that a person would attain the age of 51 years ?

30 State and establish Bayes theorem for a finite number of events.

31 (a) Define characteristic function of a random variable X. Find the characteristic function of $Y = \frac{X - \mu}{\sigma}$ in terms of the characteristic function of X (where $\mu \in R = (-\infty, \infty)$ and $\sigma > 0$).

(b) Show that the characteristic function of the sum of two independent random variables is the product of their characteristic functions.

(2 × 4 = 8 weightage)