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Name...

Reg. No.....................................

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(CUCBCSS–UG)

Core Course (Mathematics)

MAT 1B 01—FOUNDATIONS OF MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

Part A (Objective Type)

Answer **all twelve** *questions.*

1. Find the number of elements in the power set of {positive divisors of 6}.
2. Find $A \oplus B$ for $A = \{1, 2, 3\}$, $B = \{1, 2, 3\}$.
3. What is symmetric relation ?
4. Let f and g be functions defined by $f(x) = 2x + 1$ and $g(x) = x^2 - 2$ then $f \circ g$ is :
5. Find the domain of the real valued function $f(x) = \sqrt{25 - x^2}$.
6. What is the cardinal number of the set $\{\varnothing, \{\varnothing\}, \{\varnothing, \{\varnothing\}\}\}$?
7. Check whether the function $h(t) = |t^3|$ is even, odd or neither.
8. The graph of $y = x^2$ is shifted 1 unit to the right and 4 units down. Write equation of the new graph.
9. Find $\lim_{x \to \pi} \sin(x - \sin x)$.
10. State the contrapositive of the implication :

    "If it snows tonight, then I will stay at home."
11. Define a Tautology.
12. What is the truth value of $\forall x\, p(x)$ where $p(x)$ is the statement "$x^2 < 10$" and the domain consists of positive integers ?

(12 × 1 = 12 marks)

Turn over

Part B (Short Answer Type)

Answer any **nine** *questions.*

13. If A = {1, 2}, B = {a, b, c} and C = {c, d}, find (A × B) ∩ (A × C).

14. Let R be the relation R = {(1, b), (2, a), (2, c)} and S be the relation S = {(a, y), (b, x), (c, y), (c, z
Find R ∘ S.

15. Find x and y if $(y-2, 2x+1) = (x-1, y+2)$.

16. Suppose f : A → B is a constant function when will f be :

(a) one-to-one ; (b) onto.

17. Let A = {a, b} and B = {1, 2, 3}, find the number of functions from :

(a) A into B ; (b) B into A.

18. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \begin{cases} 3x-1 & \text{if } x > 3 \\ x^2-2 & \text{if } -2 \le x \le 3 \\ 2x+3 & \text{if } x < -2. \end{cases}$

Find (a) $f(-1)$; (b) $f(2)$.

19. For the function $f(x) = \begin{cases} 0 & x < 0 \\ 1 & x \ge 0 \end{cases}$ find $\lim_{x \to 0} f(x)$ or explain why they do not exist.

20. Evaluate $\lim_{x \to 4} \frac{4x - x^2}{2 - \sqrt{x}}$.

21. If $2 - x^2 \le g(x) \le 2\cos x$ for all x, find $\lim_{x \to 0} g(x)$.

22. Let p and q be the proposition "The election is decided" and "The votes have been counted" respectively. Express the compound proposition $\neg q \vee (\neg p \wedge q)$ as an English sentence.

23. Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.

24. What is the negation of the statement $\forall x \left(x^2 > x\right)$?

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*

25. Let A be a set of non-zero integers, and let ~ be a relation on A × A defined by (a,b) ~ (c,d) whenever $ad = bc$. Prove that ~ is an equivalence relation.

26. Let A = {1, 2, 3, 4}, consider the following relation R on A.

    R = {(1, 1), (2, 2), (2, 3), (3, 2), (4, 2), (4, 4)}.

    (a) Draw its directed graph ; (b) Find $R^2 = R \circ R$.

27. Consider the functions $f : A \to B$ and $f : B \to C$ prove the following :

    (a) If f and g are one-to-one then $f \circ g$ is one-to-one.

    (b) If f and g are onto functions then $g \circ f$ is an onto function.

28. Let R_1 and R_2 be relations on a set A represented by the matrix :

$$M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

Find the matrix representing (i) $R_1 \cup R_2$; (ii) $R_1 \cap R_2$.

29. Find the continuous extension to $x = 3$ of the function $f(x) = \dfrac{x^2 - 9}{x - 3}$.

30. Let $f(x) = \begin{cases} 0 & x \le 0 \\ \sin 1/x & x > 0. \end{cases}$

    (a) Does $\lim_{x \to 0^+} f(x)$ exist ? If so what is it ? If not, why not ?

    (b) Does $\lim_{x \to 0^-} f(x)$ exist ? If so what is it ? If not, why not ?

    (c) Does $\lim_{x \to 0} f(x)$ exist ? If so what is it ? If not, why not ?

31. Find the latent roots and latent vectors of the matrix $A = \begin{bmatrix} a & h & g \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$.

32. Express the statements "All lions are fierce", "some lions do not drink coffee", "some fierce creature do not drink coffee" using predicates and quantifiers, assuming the domain consists of all creatures.

33. Give a proof by contradiction of the theorem "If $3n + 2$ is odd, then n is odd".

(6 × 5 = 30 marks)

Turn over

Part D (Essay Type)

Answer any **two** *questions.*

34. Let A = {1, 2, 3, 4}. Consider the relations R and S on A given by R = {(1,1), (1,2), (2,3), (3,1), (3,3)} ; S = {(1, 2), (1, 3), (2, 1), (3, 3)}.

    Find the matrices (a) $M_{R \cap S}$; (b) $M_{R \cup S}$; (c) M_{R^c} ; (d) $M_{R \circ S}$; (e) M_{S^*} .

35. If $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$, prove that $\lim_{x \to c} (f(x) + g(x)) = L + M$.

36. Show that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent.

(2 × 10 = 20 marks)