2254 (Pages : 3) Name.....................................

Reg. No.................................

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, JANUARY 2012

(CCSS)

Mathematics—Core Course

MM 1B 01—FOUNDATIONS OF MATHEMATICS

: Three Hours Maximum Weightage : 30

Part A (Objective Type Questions)

Answer all **twelve** *questions. Each bunch of* **four** *questions carries* 1 *Weightage.*

1. Find the number of subsets of the set A = {x | x is a day of the week}.
2. If $|A| = 24$, $|B| = 69$ and $|A \cup B| = 81$, find $|A \cap B|$.
3. Find $(A \cup B) \cap (A \cup B^C)$.
4. State the DeMorgan's law of sets.

(4 × ¼ = 1 Weightage)

5. Determine whether $R = \{(1,3), (2,1)\}$ is anti-symmetric if $A = \{1,2,3,4\}$.
6. How many functions can be defined from a finite set A to a finite set B with $|A|$ and $|B|$ elements respectively?
7. Define characteristic function of a set A.
8. Whether the statement "Do not litter" is a proposition?

(4 × ¼ = 1 Weightage)

9. Write the contrapositive of the statement $p \to q$?
10. When will you say that two propositions p and q are logically equivalent?
11. Give an example of a propositional function.
12. What is the truth value of the quantification $\exists x Q(x)$ if $Q(x)$ denotes the statement $x = x + 1$ and the universe of discourse is the set of real numbers?

(4 × ¼ = 1 Weightage)

Part B (Short Answer Type Questions).

Answer all **nine** *questions.*
Each question carries 1 *Weightage.*

13. If $A = \{a, b, c, d, e\}$, $B = \{a, b, d, j, g\}$, $C = \{b, c, e, g, h\}$, find $(A \oplus C) \setminus B$.
14. Find x and y, if $(x + 2, 4) = (5, 2x + y)$.
15. Check whether the relation $x > y$ on the set of natural numbers is anti-symmetric?

Turn over

16. Define a recursive function to obtain the successive terms of the Fibonacci Series.
17. State the converse of the implication "If it snows tonight, then I will stay at home"
18. Show that $\neg(\neg p)$ and p are logically equivalent.
19. Use quantifiers to express the following statement: "Every student in this class knows to speak Hindi"
20. Determine the truth value of the statement $\exists x \forall y \neq 0 \exists (xy = 1)$ if the universe of discourse of each variable is the set of real numbers.

(9 × 1 = 9 Weig

Part C (Short Essay Questions).

Answer any **five** *questions. Each question carries* 2 *Weights.*

21. Suppose R is a partial order on a set A. Show that R^{-1} is also a partial order on A.
22. Let A be a set of nonzero integers and let ≈ be the relation on A x A defined as follows: $(a, b) \approx (c, d)$ whenever ad = bc. Prove that ≈ is an equivalence relation.
23. Suppose $\mathbb{C}$ is a collection of relations S on a set A and let T be the intersection of the relations S, i.e., $T = \cap (S : S \in \mathbb{C})$. Prove that if every S is transitive then T is transitive.
24. Consider the formula $f(x) = x^2$.
    a. Find the largest interval D such that $f: D \rightarrow R$ is a one-to-one function.
    b. Find the smallest target set T such that $f: R \rightarrow T$ is an onto function.
25. Show that $(p \vee q) \rightarrow (p \wedge q)$ is a tautology.
26. Prove that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are logically equivalent.
27. Determine whether $\exists x P(x) \wedge \exists x Q(x)$ and $\exists x (P(x) \wedge Q(x))$ are logically equivalent.

(5 × 2 = 10 Weig

Part D (Essay Questions).

Answer any **two** *questions. Each question carries* 4 *Weightage.*

28. Consider functions $f: A \rightarrow B$ and $g: B \rightarrow C$. Prove the following:
    a) If f and g are one-to-one, then $go\,f$ is one-to-one.
    b) If f and g are onto functions, then $go\,f$ is an onto function.
    c) If $go\,f$ is one-to-one, then f is one-to-one.
    d) If $go\,f$ is onto, then g is onto.

29. Consider the following five relations:

1. Relation $\leq$ (less than or equal) on the set Z of integers.
2. Set inclusion $\subseteq$ on a collection $\mathcal{C}$ of sets.
3. Relation $\perp$ (perpendicular) on the set L of lines in the plane.
4. Relation $\parallel$ (parallel) on the set L of lines in the plane.
5. Relation $|$ of divisibility on the set P of positive integers.

Determine which of the relations are: (a) reflexive, (b) symmetric, (c) anti symmetric, (d) transitive.

30. Establish the following logical equivalences where A is a proposition not involving any quantifiers.

a. $(\forall x P(x)) \vee A \Leftrightarrow \forall x (P(x) \vee A)$
b. $(\exists x P(x)) \vee A \Leftrightarrow \exists x (P(x) \vee A)$
c. $(\forall x P(x)) \wedge A \Leftrightarrow \forall x (P(x) \wedge A)$
d. $(\exists x P(x)) \wedge A \Leftrightarrow \exists x (P(x) \wedge A)$

(2 × 4 = 8 Weightage)