

**SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2018**  
(CUCBCSS-UG)

Complementary Course

MAT 2C 02—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

**Part A (Objective Type)**

*Answer all twelve questions.  
Each question carries 1 mark.*

1. Write an example for a sequence which has no upper bound.
2. Find the domain of the function  $w = xy \ln z$ .
3. Define the level surface of a function  $f$ .
4. State two path test for non-existence of limit.
5. If  $\sum_{n=1}^{\infty} |a_n|$  converges then  $\sum_{n=1}^{\infty} a_n$ .
6.  $\frac{d}{dx} \sinh x =$  \_\_\_\_\_
7. Write  $\tanh x$  in terms of exponential function.
8. Find  $\lim_{n \rightarrow \infty} \sqrt[n]{n}$ .
9.  $\int \cosh 2x =$  \_\_\_\_\_
10. Find  $\lim_{(x,y) \rightarrow (1,1)} \frac{x^2 + 2y}{3x - 2}$ .
11. Find  $\frac{\partial}{\partial x} \sin 2xy$ .
12. Define conditional convergence of a series.

(12 × 1 = 12 marks)

**Part B (Short Answer Type)**

*Answer any nine questions.  
Each question carries 2 marks.*

13. Investigate the convergence of  $\int_0^{\infty} e^{-x^2} dx$ .
14. Show that  $\lim_{n \rightarrow \infty} k = k$ , where  $k$  is a constant.

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15. Find  $\lim_{n \rightarrow \infty} \frac{\cos n}{n}$ .
16. Find  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$ .
17. Show that the function  $f(x, y) = \frac{2x^2y}{x^4 + y^2}$  has no limit as  $(x, y)$  approaches  $(0, 0)$ .
18. Find  $\frac{\partial f}{\partial y}$  if  $f(x, y) = y \sin xy$ .
19. Use chain rule to find the derivative of  $w = xy$  with respect to  $t$  along the path  $x = \cos t, y = \sin t$ . What is the derivative's value at  $t = \pi/2$ ?
20. Find the volume of the solid generated by revolving the region between the parabola  $x = y^2 + 1$  and the line  $x = 3$  about the line  $x = 3$ .
21. Show that if  $u$  is a differentiable function of  $x$  whose values are greater than 1, then
- $$\frac{d}{dx}(\cosh^{-1} u) = \frac{1}{\sqrt{u^2 - 1}} \frac{du}{dx}$$
22. Graph the sets of points whose co-ordinates satisfies the condition  $2\pi/3 \leq \theta \leq 5\pi/6$  (no restriction on  $r$ ).
23. Find a polar equation for the circle  $x^2 + (y - 3)^2 = 9$ .
24. Find the directrix of the parabola  $r = \frac{25}{10 + 10 \cos \theta}$ .

(9 × 2 = 18 marks)

**Part C (Short Essay Type)**

Answer any **six** questions.  
Each question carries 5 marks.

25. Compare  $\int_1^{\infty} \frac{dx}{x^2}$  and  $\int_1^{\infty} \frac{dx}{1+x^2}$  with limit comparison test.
26. Determine whether the series  $\sum_{n=1}^{\infty} \frac{1}{n^2}$  convergent or divergent.
27. Find the linearization of the function  $f(x, y) = x^2 + y^2 + 1$  at  $(0, 0)$ .
28. Express  $\frac{\partial w}{\partial r}$  and  $\frac{\partial w}{\partial s}$  in terms of  $r$  and  $s$  if  $w = x^2 + y^2, x = r - s$  and  $y = r + s$ .
29. Find the area of the region in the plane enclosed by the cardioid  $r = 2(1 + \cos \theta)$ .
30. Show that  $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$  if  $f(x, y, z) = e^{3x+4y} \cos 5z$ .

31. Find the Maclaurin series for the function  $f(x) = xe^x$ .
32. Does series  $\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$  convergent.
33. Find the surface area generated by revolving the curves  $x = t + \sqrt{2}$ ,  $y = \frac{t^2}{2} + \sqrt{2}t$ ,  $-\sqrt{2} \leq t \leq \sqrt{2}$  about  $y$ -axis. (6 × 5 = 30 marks)

### Part D (Essay Type)

Answer any **two** questions.  
Each question carries 10 marks.

34. Find the length of the curve  $y = \frac{1}{3}(x^2 + 2)^{3/2}$  from  $x = 0$  to  $x = 3$ .
35. Find the points of intersection of  $r^2 = 4 \cos \theta$  and  $r = 1 - \cos \theta$ .
36. Find the critical points of  $f(x) = x^{1/3}(x - 4)$ . Identify the intervals on which  $f$  is increasing and decreasing. Find the functions's local and absolute extrema values. (2 × 10 = 20 marks)