

## SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2012

(CCSS)

Mathematics—Complementary Course

MM 2C 02—MATHEMATICS

Three Hours

Maximum : 30 Weightage

Objective Type Questions. Answer *all* questions :

- 1 Find the derivative of  $\tanh \sqrt{1+t^2}$  with respect to  $t$ .
- 2 Investigate the convergence of  $\int_1^{\infty} \frac{dx}{x}$ .
- 3 Sum the series  $\sum_{n=1}^{\infty} \frac{4}{2^{n-1}}$ .
- 4 State the ratio test.
- 5 Find  $\lim_{n \rightarrow \infty} \sqrt[n]{3n}$ .
- 6 Find a formula for the  $n^{\text{th}}$  term of the sequence 0, 3, 8, 15, 24, ...
- 7 Examine the convergence of  $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ .
- 8 Graph the set of points whose polar co-ordinates satisfy the conditions  $1 \leq r \leq 2$  and  $0 \leq \theta \leq \frac{\pi}{2}$ .
- 9 Find the directrix of the hyperbola  $r = \frac{25}{10+10\cos\theta}$ .
- 10 Define the level surface of the function  $f$ .
- 11 Find  $\frac{\partial w}{\partial r}$  in terms of  $r$  and  $s$  if  $w = x^2 + y^2$ ,  $x = r - s$ ,  $y = r + s$ .
- 12 State Euler's theorem (the mixed derivative theorem).

(12 × ¼ = 3 weightage)

Turn over

II. Short Answer Type Questions. Answer *all* nine questions :

13 Use the definition of  $\cosh x$  and  $\sinh x$  to show that  $\cosh^2 x - \sinh^2 x = 1$ .

14 Show that  $\sqrt{\frac{n+1}{n}} \rightarrow 1$ .

15 Examine the convergence of  $\sum_{n=0}^{\infty} \frac{2^n + 5}{3^n}$ .

16 For what values of  $x$  does the power series  $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$  converges.

17 Find the Maclaurin's series for  $\frac{1}{1+x}$ .

18 Write the standard polar equation of an ellipse with eccentricity  $e$  and semi major

19 Find  $\frac{dy}{dx}$  if  $x^2 + \sin y - 2y = 0$ .

20 Define the linearization of a function  $f(x, y)$  at a point  $(x_0, y_0)$ .

21 Show that  $f(x, y) = \frac{2x^2y}{x^4 + y^2}$  has no limit as  $(x, y)$  approaches  $(0, 0)$ .

(9 × 1 = 9)

III. Short Essay Questions. Answer any *five* questions :

22 Show that if  $u$  is a differentiable function of  $x$ , whose values are greater than

$$\frac{d}{dx}(\cosh^{-1} u) = \frac{1}{\sqrt{u^2 - 1}} \frac{du}{dx}.$$

23 Does the sequence whose  $n^{\text{th}}$  term is  $a_n = \left(\frac{n+1}{n+1}\right)^n$  converge? If so, find  $\lim_{n \rightarrow \infty} a_n$ .

24 Find the Taylor series generated by  $f(x) = \frac{1}{x}$  at  $a = 2$ .

25 Find a polar equation for the circle  $x^2 + (y - 3)^2 = 9$ .

26 Find  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$ .

27 Find the derivative of  $f(x, y) = xe^y + \cos(xy)$  at the point  $(2, 0)$  in the direction of  $\mathbf{A} =$

28 Use the chain rule to find the derivative of  $w = xy$  with respect to  $t$  along the path  $y = \sin t$ .

(5 × 2 = 10)

V. Essay Questions. Answer any *two* questions :

- 29 Find the sum of the series  $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ .
- 30 Find the Taylor series and Taylor polynomial generated by  $f(x) = \cos x$  at  $x = 0$ .
- 31 Find the linearization  $L(x, y)$  of  $f(x, y) = x^2 - 3xy + 5$  at  $P_0(2, 1)$  and find an upper bound for  $|E|$  of the error in the approximation  $f(x, y) \approx L(x, y)$  over the rectangle  $R : |x - 2| \leq 0.1, |y - 1| \leq 0.1$ .

(2 × 4 = 8 weightage)