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Name..................................

Reg. No..................................

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2010

(CCSS Programme)

Mathematics – (Complementary Course)

MM 2C 02—MATHEMATICS

Three Hours

Maximum weightage : 30

I. Objective Type questions. (Answer *all* questions, weight $12 \times \frac{1}{4} = 3$)

1 The range of the function $y \cos hx$ is . . .

2 Give an example of a divergent sequence.

3 The n^{th} term of the sequence $1, \frac{-1}{4}, \frac{1}{9}, \frac{-1}{16}, \frac{1}{25}, \dots$ is . . .

4 The derivative of $y = \operatorname{cosec} h\,(3x)$ w.r. to x is . . .

5 The value of $\sinh^{-1} 1$ using logarithm is . . .

6 When does a sequence of real numbers $\{a_n\}$ converge to the number L ?

7 If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n \to \infty} a_n = \dots$

8 The least upper bound of the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots, \frac{n}{n+1}, \dots$ is

9 Define absolute convergence of the series $\sum a_n$.

10 The radius of the circle $r = 6 \sin \theta$ is . . .

11 The polar equation of the Cartesian equation $x = 7$ is . . .

12 State the Euler's theorem (The mixed derivative theorem).

II. Short Answer type questions. (Answer all *nine* questions, weight $9 \times 1 = 9$)

13 Find $\log_{n \to \infty} \frac{\sin n}{n}$.

14 Evaluate the integral $\int_{-\ln 4}^{-\ln 2} 2e^x \cos hx \, dx$.

15 Use partial fractions to find the sum of the series $\sum_{n=1}^{\infty} \frac{4}{(4n-3)(4n+1)}$.

16 Show that the series $\frac{1}{1} + \frac{1}{3} + \frac{1}{7} + \frac{1}{15} + \dots$ converges.

Turn over

17. Investigate the convergence of the series $\sum_{n=1}^{\infty} \frac{(2n)!}{n!n!}$.

18. Define boundary point. Give an example.

19. For what valules of x does the following power series converge ?

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots$$

20. Find the Taylor series expansion of $f(x) = e^x$ at $x = 0$.

21. State Rearrangement theorem for absolutely convergent series.

22. Find an equation for the circular cylinder $4x^2 + 4y^2 = 9$ in cylindrical coordinates.

23. Find the equation for the hyperbola with eccentricity $\frac{3}{2}$ and directrix $x = 2$.

24. Find the gradient $f(x, y) = y - x^2$ at $(-1, 0)$.

III. Short Essay questions. (Answer any *five* questions, weight $5 \times 2 = 10$)

25 Is the area under the curve $y = 1/\sqrt{x}$ from $x = 0$ to $x = 1$ finite ?

26 Evaluate $\int_2^{\infty} \frac{x+3}{(x-1)(x^2-1)} dx$.

27 Graph the lemniscate $r^2 = 4 \cos 2\theta$. What symmetries do the curves have ?

28 Find the length of the asteroid $x = \cos^3 t, y = \sin^3 t, 0 \le t \le 2\pi$.

29. Find f_x, f_y and f_z for the function $f(x, y, z) = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$.

30. Express $\partial w / \partial r$ and $\partial w / \partial s$ in terms of r and s if $w = x + 2y + z^2$, $x + r/s$, $y = r^2 + \ln$

31. Show that $f(x, y, z)$ satisfies the Laplace equation.

IV. Essay questions. (Answer any *two* questions, weight $2 \times 4 = 8$)

32. (a) Show that the alternation harmonic series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$ converges.

(b) Evaluate $\int_0^3 \frac{dx}{(x-1)^{2/3}}$.

33. (a) Find the area of the region shared by the circles $r = \cos\theta$ and $r = \sin\theta$.

(b) Find the spherical coordinate equation for the sphere $x^2 + y^2 + (z-1)^2 = 1$.

34. (a) Find the linearization L (x, y, z) of the function $f(x, y) = x^2 - xy + \frac{1}{2}y^2 + 3$.

(b) Find the local maxima, local minima, and saddle points (if any) of the function $f(x, y) = x^3 + y^3 + 3x^2 - 3y^2 - 8$.