15751 (Pages 3)

Name..................................

Reg. No..................................

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2011

(CCSS)

Complementary Course

MM 2C 02—MATHEMATICS

e : Three Hours

Maximum : 30 Weightage

I. Objective Type Questions. Answer *all* twelve questions :

1 Define hyperbolic cosine function in terms of exponential function.

2 If f is continuous on $[a, \infty)$, then, $\lim_{b \to \infty} \int_a^b f(x)\, dx =$ ————.

3 The derivative of sec h$2x$ with respect to x is = ————.

4 The value of sin h^{-1} 1 using logarithm is ————.

5 When does a sequence of real numbers $\{a_n\}$ converge to the number L.

6 The n^{th} term of the sequence 0, 1, 2, 2, 3, 3, 4, 4, is ————.

7 Find $\sum_{n=1}^{\infty} \frac{1}{9}\left(\frac{1}{3}\right)^{n-1}$.

8 The least upper bound of the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots, \frac{n}{n+1}, \ldots$ is ————.

9 Define absolute convergence of the series $\sum a_n$.

10 Define a power series about $x = a$.

11 The cylindrical co-ordinates of (0, 1, 0) (in Cartesian coordinate) is ————.

12 State the chain rule for functions of two independent variables.

(12 × ¼ = 3 weightage)

II. Short Answer questions. Answer *all* nine questions :

13 Evaluate the integral $\int \sin h2x\, dx$.

14 Use partial fractions to find the sum of the series $\sum_{n=1}^{\infty} \frac{4}{(4n-3)(4n+1)}$ converges.

Turn over

15 Show that $\log_{n \to \infty} \frac{\ln n}{n} = 0$.

16 Determine the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{2n-1}$.

17 Prove or disprove that $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$ converges.

18 State Rearrangement theorem for absolutely convergent series.

19 Define boundary point. Give an example.

20 For what values of x does the following power series converge ?

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots$$

21 Find the Taylor series expansion of $f(x) = e^x$ at $x = 0$.

(9 × 1 = 9 weight

III. Short essay questions. Answer any *five* questions :

22 Show that $\sin h^{-1} x = \ln\left(x + \sqrt{x^2+1}\right), -\infty < x < \infty$.

23 Determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n^2}\right)$.

24 State term by term differentiation theorem. Express $f(x) = \frac{1}{1-x}, |x| < 1$ as a power seri

Use the theorem to find $f'(x)$ and $f''(x)$.

25 Find the length of the asteroid $x = \cos^3 t, y = \sin^3 t, 0 \le t \le 2\pi$.

26 Graph the function $r^2 = \sin 2\theta$.

27 Find the f_x, f_y and f_z for the function $f(x, y, z) = x - \sqrt{y^2 + z^2}$.

28 Find the linearization $L(x, y, z)$ of the function $f(x, y, z) = e^x + \cos(y + z)$ at (0, 0, 0).

(5 × 2 = 10 weighta

. Essay questions. Answer any *two* questions :

29 (a) Test for absolute convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \sin n$.

(b) Identify the function :

$$f(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots ; -1 < x < 1.$$

30 (a) Find the area of the region in the plane enclosed by the cardioid $r = 2(1 + \cos\theta)$.

(b) Find the spherical co-ordinate equation for the sphere $x^2 + y^2 + (z-1)^2 = 1$.

31 (a) If $f(u, v, w)$ is differentiable and $u = x - y, v = y - z$, and $w = z - x$ then show that

$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} = 0.$$

(b) Show that $f(x, y, z) = x^2 + y^2 - 2z^2$ satisfies the Laplace equation.

(2 × 4 = 8 weightage)