C 5707 (Pages 4) Name..............................

Reg. No..............................

SECOND SEMESTER B.Sc. DEGREE (COMPUTER SCIENCE) EXAMINATION, MAY 2010

(C.C.S.S. Programme)

Statistics—Complementary Course

ST 2C 02—PROBABILITY DISTRIBUTIONS

Time : Three Hours Maximum : 30 Weightage

I. Answer all *twelve* questions :

1 $\int_{-\infty}^{\infty} f(x, y)\, dx =$ ______.

(a) $f(x)$. (b) $f(y)$.

(c) $f(x/y)$. (d) $f(x\ y)$.

2 If $f(x, y)$ is the joint p.d.f. of the r.v.s x and y, then $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x, y)\, dx dy =$ ______.

(a) $f(x).\ f(y)$. (b) $f(x)$.

(c) 0. (d) 1.

3 If X and Y are independent random variables, then $f(x/y) =$ ______.

(a) $f(x)$. (b) $f(y)$.

(c) $\frac{f(x)}{f(y)}$. (d) $f(x, y)$.

4 If $f(x, y) = 2 - x - y,\ 0 \le x \le 1\ ;\ 0 \le y \le 1$ and $f(x) = \frac{3}{2} - x, 0 < x < 1$, then $f(x/y) =$ ______.

(a) $\frac{2(2-x-y)}{3-2x}$. (b) $\frac{2(2-x-y)}{3-2y}$.

(c) $\frac{(2-x-y)(3-2x)}{2}$. (d) None.

5 If μ_{rs} is the joint r^{th} and s^{th} central moment of X and Y, then $\mu_{11} =$ ______.

(a) cov (x, y). (b) E(XY).

(c) V(X). (d) V(XY).

Turn over

6 If $\mu_{11} = 4$, $\mu_{20} = 2$ and $\mu_{02} = 10$ are the bivariate central moments of two random variables X and Y, then the regression coefficient of X on Y is ————.

(a) $\frac{1}{5}$. (b) 0.4. (c) 2. (d) 8.

7 If X has Bernoulli distribution with $p = \frac{1}{4}$, then V(X) = ————.

(a) $\frac{1}{16}$. (b) $\frac{2}{16}$. (c) $\frac{3}{16}$. (d) None.

8 If X ~ B (n, p) and if $n = 10$, $p = \frac{3}{4}$, then mode = ————.

(a) 7.5. (b) 7. (c) 8. (d) 9.

9 If X and Y are independent Poisson variates with mean 4 and 3 respectively, then P (X + Y = 0) = ————.

(a) e^{-7}. (b) 0. (c) e^{-12}. (d) e^{7}.

10 If X ~ N (μ, σ^2), then $M_x(t)$ = ————.

(a) $e^{-\mu t + \frac{t^2\sigma^2}{2}}$. (b) $e^{-\mu t + t^2\sigma^2/2}$. (c) $e^{-\mu t - \frac{t^2\sigma^2}{2}}$. (d) $e^{\mu t - \frac{t^2\sigma^2}{2}}$.

11 Which of the following statements is always true ?

(a) cov (X, Y) = 0 $\Rightarrow$ X and Y are independent.

(b) If X and Y are independent, then cov (X, Y) = 0.

(c) cov (X, Y) = 0 $\Leftrightarrow$ X and Y are independent.

(d) If X and Y are independent then cov (X, Y) may or may not be equal to zero.

12 If $\mu_{10}^1 = \frac{5}{12}$, $\mu_{01}^1 = \frac{5}{12}$ and $\mu_{11}^1 = \frac{1}{6}$ are the bivariate raw moments of two random variables X and Y, then cov (X, Y) = ————.

(a) $\frac{1}{144}$. (b) $\frac{49}{144}$. (c) $\frac{-1}{144}$. (d) 0.

(12 × ¼ = 3 weightage)

II. Short answer type questions. Answer all *nine* questions :

13 The joint probability distribution of X and Y is given below :

Y \ X	−1	1
0	$\frac{1}{8}$	$\frac{3}{8}$
1	$\frac{2}{8}$	$\frac{2}{8}$

Find the marginal probability distributions of X and Y and examine whether X and Y are independent.

14 If $P(X = x \cap Y = y) = \frac{x + 3y}{24}, \begin{matrix} x = 1, 2 \\ y = 1, 2 \end{matrix}$ find the conditional mean of X given Y = 2.

15 A r.v. X has a discrete uniform distribution over the integers 1, 2, 3, ..., n. Obtain the mean and variance of X.

16 (i) Define a Bernoulli r.v. and derive its m.g.f.

(ii) Show that sum of n independent and identically distributed Bernoulli variates is a Binomial variate.

17 Obtain the characteristic function of Poisson distribution and hence obtain its mean.

18 Define lognormal distribution.

19 Define Gamma distribution with one parameter and obtain the mean.

20 State Chebychev's inequality.

21 State Lindeberg-Levy form of CLT.

(9 × 1 = 9 weightage)

III. Short essay or Paragraph questions. Answer any *five* questions :

22 The joint p.d.f. of a two-dimensional r.v. (X, Y) is given by :

$$f(x, y) = \begin{cases} 2, & 0 < x < 1; 0 < y < x \\ 0, & \text{elsewhere} \end{cases}.$$

Find the conditional density function of X given $Y = y$.

23 If X and Y are two continuous random variables with joint p.d.f. $f(x, y)$ show that :

(a) $E[E(x/y)] = E(x)$.

(b) $V(X) = E[V(x/y)] + V[E(x/y)]$.

24 (i) Define Beta distribution of the second kind and obtain its r^{th} raw moment.

(ii) What is the mean of the distribution ?

25 Show that the central moments of Poisson distribution satisfies the recurrence relation,

$$\mu_{r+1} = r\lambda\mu_{r-1} + \frac{\lambda d}{d\lambda}\mu_r, r = 1, 2, \cdots$$

Obtain μ_2 and μ_3 from this relation.

26 Mention any *four* chief characteristics of Normal distribution.

27 If X is a r.v. such that E (X) = 3 and E (X^2) = 13, use Chebychev's inequality to determine lower bound for P (– 2 < X < 8). Also, obtain the actual probability.

28 If $\{X_n\}$ is a sequence of independent random variables such that :

$$P\left(X_n = \frac{1}{\sqrt{n}}\right) = p_n$$

$$P\left(X_n = 1 + \frac{1}{\sqrt{n}}\right) = 1 - p_n.$$

Examine whether the WLLNs is applicable to the sequence $\{X_n\}$.

(5 × 2 = 10 weightage)

IV. Essay questions. Answer *two* questions from *three* :

29 (i) Define geometric distribution and obtain its m.g.f.

(ii) Also explain the lack of memory property of geometric distribution.

30 Let : $f(x, y) = \begin{cases} 21x^2 y^3, & 0 < x < y < 1 \\ 0, & \text{elsewhere} \end{cases}$

be the joint p.d.f. of X and Y. Find the conditional mean and variance of X given Y = y.

31 A two-dimensional r.v. (X, Y) have a bivariate distribution given by :

$$P(X = x, Y = y) = \frac{x^2 + y}{32}, \quad \begin{matrix} x = 0, 1, 2, 3 \\ y = 0, 1 \end{matrix}.$$

Find :

(a) Marginal distributions and X and Y.

(b) Conditional distribution of X given Y = y.

(2 × 4 = 8 weightage)