C 41820 (Pages : 3) Name.....................................

Reg. No.....................................

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL/MAY 2013

(CCSS)

MM 2C 02—MATHEMATICS

Time : Three Hours Maximum : 30 Weightage

I. Objective type questions. Answer *all* questions, weightage ¼ each :

1 Show that $\cosh 2x = \cosh^2 x + \sinh^2 x$.

2 Investigate the convergence of $\int_0^1 \frac{1}{1-x}\,dx$.

3 Find $\lim_{n\to\infty} \frac{\ln(n^2)}{n}$.

4 Define the convergence of a sequence.

5 Find a formula for the n^{th} term of the sequence 1, – 4, 9, – 16, 25, . . .

6 Define the alternating series test.

7 The least upper bound of the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots \frac{n}{n+1}, \ldots$ is ————.

8 Graph the set of points whose polar co-ordinates satisfy the conditions $r \leq 0$ and $\theta = \frac{\pi}{4}$.

9 Show that the point $\left(2, \frac{\pi}{2}\right)$ lies on the curve $r = 2\cos 2\theta$.

10 Find $\frac{\partial t}{\partial x}$ at (4, –5) if $f(x, y) = x^2 + 3xy + y - 1$.

11 Find $\frac{\partial f}{\partial y}$ if $f(x, y) = y \sin xy$.

12 If x, y and z are independent variables and $f(x, y, z) = x\sin(y + 3z)$ find $\frac{\partial f}{\partial z}$.

(12 × ¼ = 3 weightage)

II. Short answer type questions. Answer all *nine* questions, weightage 1 each :

13 Find the derivative of $y = 2\sqrt{t}\tanh\sqrt{t}$ with respect to t.

Turn over

14 Show that the series $\sum_{n=1}^{\infty} n^2$ diverges.

15 Test the convergence of $\sum_{n=1}^{\infty} \frac{n+1}{n}$.

16 Examine the convergence of $\sum_{n=1}^{\infty} \frac{(2n)!}{n!\,n!}$.

17 For what values of x does the power series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{2n-1}}{2n-1}$ converges.

18 Find an equation for the circular cylinder $4x^2 + 4y^2 = 9$ in cylindrical co-ordinates.

19 Define the gradient of $f(x,y)$.

20 Find an equation for the tangent to the ellipse $\frac{x^2}{4} + y^2 = 2$ at (–2, 1).

21 Find $\frac{\partial z}{\partial x}$ if the equation $yz - \ln z = x + y$ defines z as a function of the two independent variables x and y and the partial derivatives exists.

(9 × 1 = 9 weightage)

III. Short essay questions. Answer any *five* questions, weightage 2 each :

22 Compare $\int_1^{\infty} \frac{dx}{x^2}$ and $\int_1^{\infty} \frac{dx}{1+x^2}$ with the limit comparison test.

23 Does $\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$ converge ?

24 Prove that $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$

25 Find the area of the region in the plane enclosed by the cardioid $r = 2(1+\cos\theta)$.

26 Verify that $W_{xy} = W_{yx}$ if $W = e^x + x\ln y + y\ln x$.

27 Find $\frac{\partial w}{\partial r}$ when $r = 1$, $s = 1$ if $W = (x+y+z)^2, x = r - s, y = \cos(r+s), z = \sin(r+s)$.

28 Find the length of the asteroid $x = \cos^3 t, y = \sin^3 t, 0 \le t \le 2\pi$

(5 × 2 = 10 weightage)

IV. Essay questions. Answer any *two* questions, weightage 4 each :

29 Show that the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \ldots + \frac{1}{n^p} + \cdots$ (p is a real number) converges if $p > 1$ and diverges if $p \leq 1$.

30 Find the Taylor series and the Taylor polynomial generated by $f(x) = e^x$ at $x = 0$.

31 Find the linearization $L(x, y)$ of $f(x, y) = e^x \cos y$ P_0 (0, 0) and find an upper bound for | E | of the error in the approximation $f(x, y) \approx L(x, y)$ over the rectangle $R : |x| \leq 0.1, |y| \leq 0.1$.

($2 \times 4 = 8$ weightage)