758 (Pages 4) Name..................................

Reg. No..................................

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2011

(CCSS)

Statistics

ST 2C 02—PROBABILITY DISTRIBUTIONS

Three Hours Maximum : 30 Weightage

Answers may be written either *in English* or *in Malayalam.*

I. Answer *all* twelve questions :

1 $\sum_{y} P(x, y) =$ ————.

(a) $P(x)$. (b) $P(y)$. (c) $P(x/y)$. (d) $P(y/x)$.

2 If $f(x) = e^{-x}, 0 \le x < \infty$, $f(y) = e^{-y}, 0 \le y < \infty$ and $f(x, y) = e^{-(x+y)}, 0 \le x, y < \infty$, then X and Y are ————.

(a) Normal variate. (b) Independent variates.

(c) Correlated variates. (d) None.

3 If the joint pmf of two random variables X and Y is :

Y \ X	0	1
0	0.28	0.37
1	0.22	0.13

then P (X = 0) = ————.

(a) 0.5. (b) 0.65. (c) 0.28. (d) 0.

4 If the p.d.f. of a r.v. X is $f(x) = \begin{cases} kx^3, & 0 \le x \le 1 \\ 0, & \text{elsewere} \end{cases}$, then $k =$ ————.

(a) 3. (b) $\frac{1}{3}$. (c) 4. (d) 1.

5 If μ^1_{rs} and μ_{rs} are the $(r, s)^{th}$ raw moments and central moments of two random variables X and Y, then $\mu_{30} =$ ————.

(a) $\mu'_{30} + 3\mu'_{20}\mu'_{10} + 2\left[\mu'_{10}\right]^3$. (b) $\mu'_{30} - 3\mu'_{20}\mu'_{10} + 2\left[\mu'_{10}\right]^3$.

(c) $\mu'_{30} - 3\mu'_{20}\mu'_{10} + 2\left[\mu'_{10}\right]^2$. (d) $\mu'_{30} - 3\mu'_{20}\mu'_{10} + 3\left[\mu'_{10}\right]^2$.

Turn over

6 If $\mu_{11} = -6, \mu_{20} = 9, \mu_{02} = 4$ are the bivariate cential moments of two random va[riables X]

and Y, then the correlation between X and Y is r_{xy} = ————.

(a) $-\frac{1}{6}$.

(b) $\frac{1}{6}$.

(c) 1.

(d) – 1.

7 If $E(X/Y) = \frac{3y}{4}$ and $E(X^2/y) = \frac{3}{5}y^2$, then V (X/y) = ————.

(a) $\frac{3}{80}y^2$.

(b) $\frac{2}{80}y^2$.

(c) $\frac{1}{80}y^2$.

(d) None.

8 If $E(X/y) = \frac{3}{4}y$, and $f_y(x) = 7y^6, 0 < y < 1$, then E(X) = ————.

(a) $\frac{16}{32}$.

(b) $\frac{15}{32}$.

(c) $\frac{12}{32}$.

(d) $\frac{21}{32}$.

9 If the m.g.f. of a Bernoulli r.v.X is $\frac{2}{5} + \frac{3}{5}e^t$, then the variance of the distribution is —

(a) $\frac{2}{5}$.

(b) $\frac{3}{5}$.

(c) $\frac{4}{25}$.

(d) $\frac{6}{25}$.

10 If X ~ B (n, p) and if $n = 20$ and $p = \frac{1}{6}$, then the distribution is ————.

(a) Positively skewed.

(b) Negatively skewed.

(c) Symmetric.

(d) Normal.

11 Which of the following is true in the case of a Poisson distribution ?

(a) Mean = Variance.

(b) Mean < Variance.

(c) Mean > Variance.

(d) None.

12 If X ~ N (μ, σ^2), then the 4^{th} central moment of X is ————.

(a) σ^4.

(b) $2\sigma^4$.

(c) $3\sigma^4$.

(d) $4\sigma^4$.

($12 \times \frac{1}{4} = 3$ w[eightage])

II. Short Answer type questions. Answer *all* nine questions :

13 A r.v. X has a discrete uniform distribution over the integers 1, 2, 3, ..., n. Obtain the m.g.f. of X and hence obtain the mean.

14 Define Binomial distribtion and obtain the mean.

15 Obtian the cumulant generating function of Poisson distribution and hence obtain the first 4 cumulants.

16 Define Pareto distribution.

17 Define continuous rectangular distribution in the interval (0, 2) and hence obtain the m.g.f.

18 Two random variables X and Y have the joint p.d.f. :

$$f(x, y) = \begin{cases} 6x^2 y, & 0 < x < 1, 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}.$$

Find the conditional mean of Y given $X = x$.

19 The joint probability distribution of X and Y is given by :

Y \ X	1	2	3
1	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$
2	$\frac{1}{4}$	$\frac{1}{4}$	0
3	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$

Find the marginal probability distribution of X and Y and examine whether X and Y are independent.

20 State Bernoulli law of large Numbers.

21 Define convergence in probability.

(9 × 1 = 9 weightage)

III. Short essay or paragraph questions. Answer any *five* questions :

22 The joint p.d.f. of a two-dimensional r.v. (X, y) is given by :

$$f(x, y) = \begin{cases} x + y, & 0 \le x \le 1 ; 0 \le y \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find the conditional density function of X given $Y = y$.

23 Define Beta distribution of the 1st kind and obtain the harmonic mean.

24 Show that the central moments of Binomial distirbution statisfies the recurrence relation

$$\mu_{r+1} = pq\left(nr\,\mu_{r-1} + \frac{d}{dp}\mu_r\right), r = 1, 2, 3, ...$$

25 If $X_1, X_2, ..., X_n$ are independent normal variates with mean $\mu_1, \mu_2, ... \mu_n$ and varian $\sigma_1^2, \sigma_2^2, ..., \sigma_n^2$ respectively, show that :

$$\sum a_i X_i \sim N\left[\sum a_i \mu_i, \sum a_i^2 \sigma_i^2\right]$$

Hence show that $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$.

26 A r.v.X has the density function, $f(x) = e^{-x}, 0 \le x < \infty$. Show that the Chebyschev's inequal gives $P\{|x-1| > 2\} < \frac{1}{4}$ and show that the actual probability is e^{-3}.

27 Examine whether the WLLNs holds good for the sequence $\{X_n\}$ of independent r.v.s, wher

$$P\left(X_n = \frac{1}{\sqrt{n}}\right) = \frac{2}{3}$$

$$P\left(X_n = \frac{-1}{\sqrt{n}}\right) = \frac{1}{3}.$$

28 Let : $f(x, y) = e^{-(x+y)}, \begin{matrix} 0 < x < \infty \\ 0 < y < \infty \end{matrix}$

Find cov (X, y).

(5 × 2 = 10 weighta

IV. Essay Type questions. Answer any *two* questions :

29 Let : $f(x, y) = \begin{cases} 8xy, & 0 < x < y < 1 \\ 0, & \text{elsewhere} \end{cases}$

Find var (Y/X = x).

30 The joint probability distribution of X and Y is given by :

Y \ X	− 1	1
0	1/8	3/8
1	2/8	2/8

Find the :

(i) correlation coefficient between X and Y.

(ii) Regression coefficient of X on Y.

(iii) Regression coefficient of Y on X.

31 Define exponential distribution and obtain its m.g.f. Also explain the lack of memory proper of exponential distribution.

(2 × 4 = 8 weighta