D 31897 (Pages : 3) Name.....

Reg. No.....

SECOND SEMESTER B.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION DECEMBER 2012

(CCSS)

MM 2C 02—MATHEMATICS

Time : Three Hours Maximum : 30 Weightage

I. Objective Type Questions : (Answer *all* questions, weight 12 × ¼ = 3)

1. Integrate $\coth 5x$.

2. Investigate the convergence of $\int_1^\infty \frac{dx}{x^2}$.

3. Find $\lim_{n\to\infty} \frac{2^n}{5n}$.

4. Define Absolute convergence.

5. Find a formula for the n^{th} term of the sequence 1, –1, 1, –1, 1, ...

6. State the Sandwich theorem for sequences.

7. Examine the convergence of $\sum_{n=1}^{\infty} \frac{2^n}{n^2}$.

8. Graph the set of points whose polar co-ordinates satisfy the conditions $-3 \le r \le 2$ and $\theta = \frac{\pi}{4}$.

9. Find the polar equation of the hyperbola with eccentricity $\frac{3}{2}$ directrix $x = 2$.

10. Define the continuity at (x_0, y_0) of the function $f(x, y)$.

11. Find $\frac{\partial^2 w}{\partial x \partial y}$ if $w = xy + \frac{e^y}{y^2 + 2}$.

12. Find f_x if $f(x, y) = \frac{2y}{y + \cos x}$.

Turn over

II. Short Answer Type Questions : (Answer all *nine* questions, weight 9 × 1 = 9)

13. Evaluate $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$.

14. Examine the convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n 5}{4^n}$.

15. Does the series $5+\frac{2}{3}+1+\frac{1}{7}+\frac{1}{2}+\frac{1}{3!}+\frac{1}{4!}+\ldots+\frac{1}{k!}+\ldots..$ converge ?

16. Examine the convergence of $\sum_{n=1}^{\infty} \frac{4^n n!n!}{(2n)!}$.

17. For what values of x does the power series $\sum_{n=0}^{\infty} n!x^n$ converges.

18. Find the distance from one focus of the ellipse with semimajor axis 39.44 AU and eccentricity 0.25, to the associated directrix.

19. Find $\left(\frac{\partial w}{\partial x}\right)_{y,z}$ if $w = x^2 + y - z + \sin t$ and $x + y = t$.

20. Find $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x^2 + y^2$, $x = r - s$, $y = r + s$.

21. If Resistors of R_1, R_2 and R_3 ohms are connected in parallel to make an R-ohm resistor, the value of R can be found from the equation $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$. Find the value of $\frac{\partial R}{\partial R_2}$ where $R_1 = 30$, $R_2 = 45$, $R_3 = 90$ ohms.

III. Short Essay Questions : (Answer any *five* questions, weight 5 × 2 = 10)

22. Evaluate $\int_0^1 \frac{2dx}{\sqrt{3+4x^2}}$.

23. Sum the series $\sum_{n=1}^{\infty} \frac{3^{n-1}-1}{6^n-1}$.

24. Prove that $\tanh^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots.$

25. Graph the cardioid $r = 1 - \cos\theta$.

26. The plane $x = 1$ intersects the paraboloid $z = x^2 + y^2$ in a parabola. Find the slope of the tangent to the parabola at $(1, 2, 5)$.

27. Find $\frac{dw}{dt}$ if $w = xy + z,\ z = \cos t,\ y = \sin t,\ z = t$.

28. If $z = x + f(u)$ where $u = xy$ show that $x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = x$.

IV. Essay Questions : (Answer any *two* questions, weight $2 \times 4 = 8$)

29. Does the series $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \ldots + \frac{1}{n^2} + \ldots$ converge ?

30. Multiply the geometric series $\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \ldots + x^n + \ldots = \frac{1}{1-x}$, for $|x| < 1$, by itself to get a power series for $\frac{1}{(1-x)^2}$.

31. Find the linearization $L(x, y)$ of $f(x, y) = 1 + y + x\cos y$ at $P_0(0, 0)$ and find an upper bound for $|E|$ of the error in the approximation $f(x, y) \approx L(x, y)$ over the rectangle $R : |x| \le 0.2,\ |y| \le 0.2$.