**C 41828** (Pages : 4) Name.................................

Reg. No.................................

# SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL/MAY 2013

(CCSS)

Statistics

## ST 2C 02—PROBABILITY DISTRIBUTIONS

Time : Three Hours Maximum : 30 Weightage

I. Objective type questions. Answer *all* twelve questions :

1. The joint probability mass function (p.m.f.) of a bivariate discrete random variable (X, Y) is :

   (a) $P(X \leq x, Y \leq y)$. (b) $P(X \geq x, Y \geq y)$.

   (c) $P(X \leq x, Y = y)$. (d) $P(X = x, Y = y)$.

2. The conditional p.m.f. of a discrete random variable X given a discrete random variable $Y = y$ is undefined when :

   (a) $P(Y = y) > 0$. (b) $P(Y = y) = 0$.

   (c) $P(X = x) > 0$. (d) $P(X = x) = 0$.

3. If X and Y are independent random variables with finite expectations, then E (XY) is :

   (a) $= E(X) \cdot E(Y)$. (b) $\leq E(X) \cdot E(Y)$.

   (c) $\geq E(X) \cdot E(Y)$. (d) $\leq [E(X) \cdot E(Y)]^{\frac{1}{2}}$.

4. If $\mu_{rs}$ denote the $(r, s)^{th}$ product central moment of a bivariate random variable (X, Y), then which of the following is always true ?

   (a) $\mu_{00} = 0$. (b) $\mu_{10} = \mu_{01}$.

   (c) $\mu_{20} = \mu_{02}$. (d) $\mu_{11} = 0$.

5. $E\{E(X|Y)\}$ is always :

   (a) $> E(X|Y)$. (b) $= E(X|Y)$.

   (c) $> E(X)$. (d) $= E(X)$.

6. If X follows binomial distribution B (8, 0.4), then the distribution of Y = 8–X is :

   (a) B (8, 0.4). (b) B (4, 0.4).

   (c) B (8, 0.6). (d) B (4, 0.6).

**Turn over**

7. If X and Y are independent Poisson variates such that X~P(2) and Y~P(1), then the distribution of X – Y is :

   (a) P(1).
   (b) P(2).
   (c) P(3).
   (d) None of these.

8. Variance of a degenerate random variable X degenerate at a positive real number C is :

   (a) Positive.
   (b) Unity.
   (c) Zero.
   (d) Equal to C.

9. In case of normal distribution $N(\mu, \sigma)$, the maximum probability occurring at the point $x = \mu$ is :

   (a) $\frac{1}{\sqrt{2\pi}}$.
   (b) $\frac{1}{\sigma}$.
   (c) $\frac{1}{\sqrt{2\pi\sigma}}$.
   (d) $\frac{1}{\sigma\sqrt{2\pi}}$.

10. Seventh central moment of $N(\mu, \sigma)$ is :

    (a) Zero.
    (b) One.
    (c) $\sigma^7$.
    (d) $\mu^7 + 3\sigma^7$.

11. In case of one parameter gamma distribution :

    (a) Mean > Variance.
    (b) Mean < Variance.
    (c) Mean = Variance.
    (d) Mean $= \frac{1}{\text{Variance}}$.

12. The income of people exceeding a certain limit follows :

    (a) Cauchy.
    (b) Pareto.
    (c) Beta.
    (d) Log-normal.

$(12 \times \frac{1}{4} = 3$ weightage)

II. Short answer type questions. Answer *all* nine questions :

13. Define marginal probability function.
14. Define conditional density function.
15. Define conditional mean of X given $Y = y$, in continuous case.
16. Define conditional covariance.
17. Define discrete uniform distribution.

18. Find the moment generating function of Bernoulli distribution.
19. State the cumulative distribution function of rectangular distribution over $(-a, a)$.
20. Define log-normal distribution.
21. State Bernoulli's law of large numbers.

(9 × 1 = 9 weightage)

III. Short Essay or Paragraph questions. Answer any *five* questions :

22. If $f(x, y) = \begin{cases} k\,x^2 y, & 0 < x < 1\,; 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$ is a joint density function of (X, Y), find the value of $k$ ?

23. Let $f(x, y) = \begin{cases} \frac{1}{8}(6 - x - y), & 0 \le x < 2 \;;\; 2 \le y < 4 \\ 0, & \text{elsewhere} \end{cases}$. Find $P(X < 1 \mid Y < 3)$.

24. Let $f(x, y) = \begin{cases} 2, & 0 < x < 1\,; 0 < y < x \\ 0, & \text{elsewhere} \end{cases}$ be the joint probability density function of (X, Y). Verify whether X and Y are independent.
25. Derive Poisson distribution as a limiting case of binomial distribution.
26. Find the mode of binomial distribution for which mean is 4 and standard deviation $\sqrt{3}$.
27. Derive the moment generating function of gamma distribution and hence obtain its mean and variance.
28. Obtain the harmonic mean of beta distribution of second kind.

(5 × 2 = 10 weightage)

IV. Essay questions. Answer any *two* questions :

29. Let (X, Y) has joint probability density function $g(x, y) = \begin{cases} x\,e^{-x(1+y)}, & x > 0, y > 0 \\ 0, & \text{elsewhere} \end{cases}$.

Find :

(a) E (Y),

(b) E (XY) and

(c) $E(Y \mid X = x)$.

30. (a) Derive normal distribution as a limiting form of binomial distribution.

    (b) Derive the cumulant generating function of normal distribution and hence obtain its first four cumulants.

31. (a) State and establish Chebychev's inequality.

    (b) Discuss central limit theorem and its applications.

(2 × 4 = 8 weightage)