D 31904 (Pages : 4) Name..............................

Reg. No..............................

SECOND SEMESTER B.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION DECEMBER 2012

(CCSS)

Statistics

ST 2C 02—PROBABILITY DISTRIBUTIONS

Time : Three Hours Maximum : 30 Weightage

I. Objective type questions. Answer all *twelve* questions :

1. If (X, Y) is a bivariate discrete random variable, then $\sum_{y} P(X \leq x, Y = y) =$

   (a) $P(X = x)$. (b) $P(X < x)$.

   (c) $P(X \leq x)$. (d) None of these.

2. For a bivariate continuous random variable (X, Y), $P(a_1 < X < a_2,\ b_1 < Y < b_2)$ is :

   (a) $< P(a_1 < X \leq a_2,\ b_1 < Y \leq b_2)$.

   (b) $= P(a_1 < X \leq a_2,\ b_1 < Y \leq b_2)$.

   (c) $< P(a_1\, a_2 < XY \leq b_1\, b_2)$.

   (d) $= P(a_1\, a_2 < XY \leq b_1\, b_2)$.

3. If X and Y are independent discrete random variables, then $P(X \leq x, Y \leq y)$ is :

   (a) $\leq P(X = x, Y = y)$. (b) $\leq P(X \leq x) \cdot P(Y \leq y)$.

   (c) $= P(X = x) \cdot P(Y = y)$. (d) $= P(X \leq x) \cdot P(Y \leq y)$.

4. In case of a bivariate random variable (X, Y) with finite product central moments μ_{rs} of order (r, s), the $\text{cov}(X, Y)$ is :

   (a) μ_{11}. (b) μ_{22}.

   (c) $\mu_{22} + \mu_{02} \cdot \mu_{20}$. (d) $\mu_{22} - \mu_{02} \cdot \mu_{20}$.

Turn over

5. $E\left[\text{Var}(X|Y)\right]=$

(a) $\text{Var}(X)$. (b) $\text{Var}(X)+\text{Var}\left[E(X|Y)\right]$.

(c) $\text{Var}(X)-\text{Var}\left[E(X|Y)\right]$. (d) $\text{Var}(X)-E\left[E(X|Y)\right]$.

6. In case of Bernoulli distribution :

(a) Mean = Variance. (b) Mean < Variance.

(c) Mean > Variance. (d) Mean ≤ Variance.

7. If X and Y are independent Poisson variates each with mean 3, then $Z = X + Y$ follows :

(a) Poisson with mean 3. (b) Poisson with mean 6.

(c) Poisson with mean 9. (d) None of these.

8. If X follows geometric distribution with $p=\frac{1}{3}$, then $P(X\geq 2)=$

(a) $\frac{1}{3}$. (b) $\frac{2}{3}$.

(c) $\frac{1}{9}$. (d) $\frac{2}{9}$.

9. The mean of standard normal distribution is :

(a) Zero. (b) Unity.

(c) Positive. (d) Not finite.

10. As sample size becomes large, most of the distributions occurring in practice tend to :

(a) Exponential. (b) Normal.

(c) Log-normal. (d) Cauchy.

11. If X follows log-normal distribution, the value of $P(X=0.25)$ is :

(a) 0.25. (b) 0.5.

(c) Zero. (d) One.

12. If X follows beta type 1 $\beta_1(p,q)$, the distribution of $Y=1-X$ is :

(a) $\beta_1(p,q)$. (b) $\beta_1(q,p)$.

(c) $\beta_2(p,q)$. (d) $\beta_2(q,p)$.

(12 × ¼ = 3 weightage)

II. Short answer type questions. Answer all *nine* questions :–

13. Define conditional probability function.
14. Define stochastic independence of random variables.
15. Define conditional expectation.
16. Find the characteristic function of degenerate distribution.
17. State the lack of memory property of geometric distribution.
18. Define rectangular distribution over (a, b).
19. State additive property of gamma distribution.
20. Define Pareto distribution.
21. State Chebychev's inequality.

(9 × 1 = 9 weightage)

III. Short Essay or Paragraph questions. Answer any *five* questions.

22. If $P(X = x, Y = y) = k(x^2 + y)$, for $x = 0, 1, 2, 3$ and $y = 0, 1$, find the value of k?

23. Let $f(x, y) = \begin{cases} 6x^2 y, & 0 < x < 1,\ 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$

    be the joint probability density function of (X, Y). Find $P(X > Y)$.

24. If joint cumulative distribution function of X and Y is :

    $$F(x, y) = \begin{cases} 1 - e^{-x} - e^{-y} + e^{-(x+y)}, & x > 0, y > 0 \\ 0, & \text{elsewhere} \end{cases}$$

    Examine whether X and Y are independent.

25. Define discrete uniform distribution over $[1, n]$. Obtain its mean and variance.
26. Obtain mode of Poisson distribution.
27. Derive the quartile deviation of normal distribution.
28. State and establish Bernoulli's law of large numbers.

(5 × 2 = 10 weightage)

IV. Essay questions. Answer any *two* questions :

29. Let (X, Y) has probability density function $g(x, y) = \begin{cases} 21x^2y^3, & 0 < x < y < 1 \\ 0, & \text{elsewhere} \end{cases}$

    Obtain the conditional mean and conditional variance of X given $Y = y$.

30. (a) Derive the moment generating function of exponential distribution and hence obtain its mean and variance.

    (b) Define beta distribution of first kind. Obtain its mean and variance.

31. (a) Explain convergence in probability.

    (b) State and establish a weak law of large numbers for independent and identically distributed random variables.

($2 \times 4 = 8$ weightage)