

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2010
(CCSS)

Mathematics (Complementary Course)

MM 3C 03—MATHEMATICS

Time : Three Hours

Maximum Weightage : 30

I. Objective Type questions

(Answer all questions, weightage $12 \times \frac{1}{4} = 3$)

1. The order of the differential equation $(y')^2 + y = x^2 - 2$ is ...
2. The variable separable equation $g(y)dy = f(x)dx$ has the general solution ...
3. The necessary and sufficient condition for a differential equation $M(x, y)dx + N(x, y)dy = 0$ is exact is that ...
4. The rank of a zero matrix is ...
5. The system of linear equations $AX = B$ is consistent if and only if ... have the same rank
6. The roots of the equation $|A - \lambda I| = 0$ is called ...
7. The work done by a constant force p in making a displacement d is given by ...
8. If the scalar triple product of three non zero vectors is zero then those vectors are ...
9. The gradient of a scalar point function is a vector whose direction is ...
10. $\text{div}(\text{curl } \mathbf{v}) = \dots$
11. The line integral $\int_C \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r}$ is path independent if and only if ... is a zero vector.
12. By Gauss divergence theorem, $\iiint_T \text{div } \mathbf{F} dV = \dots$ where T is a region in space

II. Short answer type questions

(Answer all questions, weightage $9 \times 1 = 9$)

13. Solve the initial value problem

$$y' + y = 0; y(0) = 3$$

14. Find the integrating factor of the differential equation $\frac{dy}{dx} + y \log x = \frac{x}{3}$

15. Obtain the row equivalent canonical matrix C of $\begin{bmatrix} 1 & 3 & 4 \\ 2 & 6 & 8 \end{bmatrix}$

16. Test for consistency of the equations:

$$2x + 3y = 13$$

$$5x - 2y = 4$$

17. Find the eigen values of $A = \begin{bmatrix} -2 & -1 \\ 5 & 4 \end{bmatrix}$

18. Find the resultant (in components) and its magnitude of $\mathbf{p} = [4, -2, -3]$, $\mathbf{q} = [8, 8, 1]$,
 $\mathbf{u} = [-12, -6, 2]$

19. A force $\mathbf{p} = [3, 0, -6]$ acts on a line through a point $[0, -1, 4]$. Find the moment \mathbf{m} of \mathbf{p} about the point $[4, 6, -1]$

20. Find the directional derivative of $f(x, y, z) = 2x^2 + 3y^2 + z^2$ at the point $P(2, 1, 3)$ in the direction of the vector $i - 2k$

21. Find the curl of the vector $\mathbf{v} = yz\mathbf{i} + 3zx\mathbf{j} + z\mathbf{k}$

III. Short Essay or Paragraph Questions

Answer any 5 questions from 7 (5X2 = 10 weightage)

22. Solve the initial value problem: $y' + y \tan x = \sin 2x$; $y(0) = 1$

23. Find the orthogonal trajectories of $y = cx^2$ where c is arbitrary.

24. Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{bmatrix}$

25. Determine the eigen vectors of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

26. If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$, find A^2 , using Cayley Hamilton theorem

27. Evaluate the integral $I = \int_C 3x^2 dx + 2yzdy + y^2 dz$ from $A(0, 1, 2)$ to $B(1, -1, 7)$

28. Using Gauss Divergence theorem, evaluate $\iint_S x^3 dydz + x^2 y dzdx + x^2 dx dy$
 where S is the closed surface consisting of the cylinder $x^2 + y^2 = a^2$ ($0 \leq z \leq b$) and the circular disks $z = 0$ and $z = b$ ($x^2 + y^2 \leq a^2$)

IV. Essay Questions

Answer any 2 questions from 7 (2X4 = 8 weightage)

29. Solve: $x + 3y - 2z = 0$, $2x - y + 4z = 0$, $x - 11y + 14z = 0$
30. Using Green's theorem evaluate $\oint_C (xy + y^2)dx + x^2 dy$ where C is bounded by $y = x$ and $y = x^2$
31. Verify Stoke's theorem for $F = [y, z, x]$ and S the paraboloid $z = f(x, y) = 1 - (x^2 + y^2)$, $z \geq 0$