1095 (Pages : 3) Name.................................

Reg. No.................................

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2012

Mathematics—Complementary Course

MM 3C 03—MATHEMATICS

: Three Hours Maximum : 30 Weightage

Answer *all* questions. Each question of weightage ¼).

1 When is M $(x, y)\, dx$ + N $(x, y)\, dy$ an exact differential equation ?

2 What is the Bernoulli equation ?

3 Solve : $y'' = x^{-4}$.

4 Define rank of a non-zero matrix 'A'.

5 Are the matrices $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ and $\begin{bmatrix} 2 & 4 \\ 3 & 3 \end{bmatrix}$ equivalent ?

6 What are the characteristic roots of $A = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ 9 & -5 & 3 \end{bmatrix}$?

7 What is the divergence of $\vec{a} = \left[3xz, 2xy, -yz^2\right]$?

8 What is the volume of a parallel piped with edge vectors $\vec{a}, \vec{b}$ and $\vec{c}$?

9 State Laplace's equation.

10 If a surface S is given by $g(x, y, z) = 0$, what is the unit normal vector to S ?

11 Give the parametric representation of the sphere $x^2 + y^2 + z^2 = a^2$.

12 Give an example of a non-orientable surface.

(12 × ¼ = 3 weightage)

Answer *all* questions. Each question of weightage 1.

13 Solve : $(1 - y)\dfrac{dy}{dx} = 1 + x.e^{x^2}, y(0) = 1$.

14 Find an integrating factor for $\left(2\cos y + 4x^2\right)dx = x\sin y\, dy$.

15 Find the rank of $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 3 & 7 & 8 \end{bmatrix}$.

Turn over

16 If $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$, use Cayley Hamilton theorem to find A^4.

17 If $\vec{a} = [1,1,0], \vec{b} = [3,2,1]$ and $\vec{c} = [1,0,2]$, find the angle between $\vec{a}$ and $\vec{b} + \vec{c}$

18 Find the tangential and normal accelerations of $\vec{r}(t) = 5t^2\hat{k}$.

19 Prove that curl (grad f) $= \vec{0}$.

20 Check for path independence : $3z^2dx + 6xzdz$.

21 Use Green's theorem to find the area enclosed by the circle $x^2 + y^2 = 9$.

(9 × 1 = 9 wei

III. Answer any *five* questions from seven. Each question of weightage 2.

22 Solve : $(2x - 4y + 5)y' + (x - 2y + 3) = 0$.

23 Find the rank by reducing to normal form :

$$A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}.$$

24 Find the eigenvalues of $A = \begin{bmatrix} -2 & -8 & -12 \\ 1 & 4 & 4 \\ 0 & 0 & 1 \end{bmatrix}$.

25 Find the directional derivative of $f = xyz$ along [1, –2, 2) at (–1, 1, 3).

26 Test for exactness and hence evaluate :

$$\int_{(0,0,0)}^{(a,b,c)} 2xy^2dx + 2x^2ydy + dz.$$

27 Find the length of the catenary $\vec{r}(t) = t\,\hat{i} + \cosh t\,\hat{j}$ from $t = 0$ to $t = 1$.

28 Evaluate $\iint_S \vec{F} \cdot \vec{n}\, dA$ using the Divergence theorem, where :

$\vec{F} = [x^2, 0, z^2]$ and S is the box $|x| \le |, |y| \le 3, |z| \le 2$.

(5 × 2 = 10 weig

Answer any *two* questions. Each question of weightage 4.

29 Find the Orthogonal trajectories of $y = c \cdot e^{-x}$.

30 Verify Cayley Hamilton theorem for :

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}.$$

31 Verify Stokes theorem for $\bar{F} = [z^2, 5x, 0]$ and S is the square $0 \le x \le 1, 0 \le y \le 1, z = 1$.

(2 × 4 = 8 weightage)