D 51294 (Pages : 4) Name.................................

Reg. No.................................

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(CUCBCSS—UG)

Complementary Course

STS 3C 03—STATISTICAL INFERENCE

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions each in* **one word**.
Each question carries 1 mark

Name the following :—

1. The probability distribution of the sample mean of 16 random samples taken from a normal population with mean 5 and SD 4.
2. The probability distribution of the ratio of two independent standard normal random variables.
3. The value of a statistic representing the value of a population parameter.

Fill up the blanks :

4. The interval for the value of an unknown parameter with a specified probability is called ———.
5. ——— distribution is derived as the ratio of two independent Chi-square random variables.
6. In a statistical testing of hypothesis, the hypothesis is to be tested is termed as ———.
7. The rejection region in testing of hypothesis is called ———.

Write True *or* False :

8. If T is an unbiased estimator of θ, then $E(T) = \theta^2$.
9. F-test is used to test the equality of variances of two normal populations.
10. In a testing procedure, type II error is more serious.

(10 × 1 = 10 mar

Section B

Answer **all** *questions in* **one sentence** *each.*
Each question carries 2 marks.

11. Define Sampling Distribution.
12. Define Statistical Inference.

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13. Show that p.d.f. of exponential distribution with parameter ½ and Chi-square distribution with 2 d.f. are same.

14. A random sample of size 16 is taken from a normal population with mean 30 and variance 64. Find the probability that the sample variance S^2 will be less than the population variance.

15. Let $\bar{X}$ be the mean of n random samples taken from $N(\mu, \sigma)$ and S^2 be the sample variance.

    Establish that $\frac{(\bar{x}-\mu)\sqrt{n-1}}{S} \sim t_{(n-1)}$.

16. Define most powerful test.

17. For the random sample $x_1, x_2, \ldots, x_n$ taken from Poisson population with parameter λ, show that $\frac{n\bar{x}}{n+1}$ is a biased estimator λ.

(7 × 2 = 14 marks)

Section C

Answer any **three** *questions.*
Each question carries 4 marks.

18. Obtain the mean and variance of a Chi-square random variable with n degrees of freedom.

19. Let x_1 and x_2 denote random samples from a normal population with mean θ and variance unity. Show that $y_1 = x_1 + x_2$ is a sufficient statistic for θ.

20. Explain the method of moment estimation.

21. Distinguish between point and interval estimation.

22. Define size and power of a test in testing of hypothesis.

(3 × 4 = 12 mark

Section D

Answer any **four** *questions.*
Each question carries 6 marks.

23. Define Students t-distribution. If X_1 and X_2 are two independent standard normal variable prove that $t = \frac{\sqrt{2}\,X_1}{\sqrt{X_1^2 + X_2^2}}$ follow t-distribution with 2 d.f.

24. If X is a random variable following F distribution with (n_1, n_2) degrees of freedom. Prove that the distribution of $Y = \frac{1}{X}$ is F distribution with (n_2, n_1) degrees of freedom.

25. Obtain the MLE of α and β using the random samples $x_1, x_2, ..., x_n$ taken from the population with p.d.f. $f(x) = \frac{1}{\beta} e^{-\frac{(x-\alpha)}{\beta}}, x \geq \alpha, \beta > 0$.

26. Estimate a 95 % confidence interval for μ, based on 10 random samples

22, 25, 30, 21, 24, 26, 24, 28, 25, 26 taken from N $(\mu, 5)$.

27. Hemoglobin levels of children under age 6 are distributed as normal population N $(\mu, 0.85)$. To test $H_0 : \mu = 12.3g/100\ ml$ against $H_A : \mu = 11.5g/100\ ml$. It is decided to reject null hypothesis, if $\bar{x} \leq 11.8$, where $\bar{x}$ is the sample mean of 25 samples. Find significance level and power of the test.

28. Explain the small sample test to test the mean of a normal population when σ is unknown.

(4 × 6 = 24 marks

Section E

Answer any **two** *questions.*
Each question carries 10 *marks.*

9. (i) Derive the sampling distribution of means of samples taken from a normal population N (μ, σ)

(ii) A random sample of size 25 is taken from a normal population with mean 1 an variance 9. What is the probability that the sample mean is negative ?

. x_1, x_2 are two random sample taken from a population with p.d.f. $f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}, 0 < x < \infty; \theta >$

To test $\theta = 2$ against $\theta = 4$, the critical region is $x_1 + x_2 \geq 9.5$. Obtain the significance level a power of the test.

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31. (a) Explain Chi-square test of independence.

(b) The following table gives the classification of 100 workers according to sex, and the nature of work. Test whether nature of work is independent of the sex of the worker at 5 % level of significance.

		Skilled	*Unskilled*	*Total*
Male	..	40	20	60
Female	..	10	30	40
Total	..	50	50	100

32. (a) Explain F-test of equality of variances of two normal populations.

(b) Following are the set of observations from two normal populations. Test the equality of their population variances at 5 % of significance level :

From first population	:	39	41	43	41	45	39	42	44
From second population	:	40	42	40	44	39	38	40	

(2 × 10 = 20 marks)