

## THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(CUCBCSS—UG)

Core Course

MAT 3B 03—CALCULUS AND ANALYTIC GEOMETRY

Time : Three Hours

Maximum : 80 Marks

## Part A (Objective Type)

Answer all twelve questions.

1. Find  $\frac{d}{dx} \ln 2x$ .
2. Define a sequence.
3. Find least upper bound of  $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots, \frac{n}{n+1}$ .
4. Find a formula for  $n^{\text{th}}$  term of the sequence 1, 5, 9, 13, 17, ....
5. State Sandwich theorem for sequences.
6. If  $|r| < 1$  the series  $a + ar + ar^2 + \dots + ar^{n-1} + \dots$  converges to.....
7. Define conditional convergence of a series.
8. Write a parametrization of the circle  $x^2 + y^2 = 1$ .
9.  $\lim_{n \rightarrow \infty} \sqrt[n]{n} = \dots\dots\dots$
10. Write the polar form of the parabola  $y^2 = Qax$ .
11. Suppose that  $a_n > 0$  and  $b_n > 0$  for all  $n \geq N$ . If  $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \infty$  and  $\sum b_n$  diverges, then  $\sum a_n \dots\dots\dots$
12. If  $\sum |a_n|$  is convergent, then  $\sum a_n$  is .....

(12 × 1 = 12 marks)

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Part B (Short Answer Type)

Answer any nine questions.

13. Find  $\int_{-\pi/2}^{\pi/2} \frac{4 \cos \theta}{3 + 2 \sin \theta} d\theta$ .

14. Find  $k$  if  $e^{2k} = 10$ .

15. Find  $\int_0^{\ln 2} e^{3x} dx$ .

16. Show that  $\lim_{n \rightarrow \infty} \frac{1}{n} = 0$ .

17. For what values of  $x$  do the power series  $\sum_{n=0}^{\infty} \frac{x^n}{n!}$  converge?

18. Find the series for  $f'(x)$  and  $f''(x)$  if  $f(x) = \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, -1 < x < 1$ .

19. Find the focus and directrix of the parabola  $y^2 = 10x$ .

20. Find the eccentricity of the hyperbola  $x^2 - y^2 = 1$ .

21. Determine the conic section from the equation  $xy - y^2 - 5y + 1 = 0$ .

22. Graph the sets of points whose polar co-ordinates satisfy the conditions  $-3 \leq r \leq 2$  and  $\theta =$

23. Replace the polar equation  $r^2 = 4r \cos \theta$  by equivalent Cartesian equation.

24. Find the equation for the hyperbola with eccentricity  $3/2$  and directrix  $x = 2$ .

(9 × 2 = 18)

## Part C (Short Essay Type)

Answer any six questions.

25. Solve the initial value problem  $e^x \frac{dy}{dx} = 2x, x > \sqrt{3}, y(2) = 0$ .
26. Show that  $(-1)^{n+1} \frac{n-1}{n}$  diverges.
27. Find a formula for the  $n^{\text{th}}$  partial sum of the series  $\frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \dots + \frac{1}{(n+1)(n+2)} + \dots$  and use it to find the series sum if it converges.
28. Identify the function  $f(x) = x + \frac{x^3}{3} + \frac{x^5}{5} + \dots, -1 \leq x \leq 1$ .
29. The  $x$  and  $y$  axes are rotated through an angle of  $\pi/4$  radians about the origin. Find an equation for the hyperbola  $2xy = 9$  in the new co-ordinates.
30. Find the surface area generated by revolving the curves  $x = \cos t, y = 2 + \sin t, 0 \leq t < 2\pi$  about  $x$ -axis.
31. Show that  $(1/2, 3\pi/2)$  lies on the curve  $r = -\sin(\theta/3)$ .
32. Determine whether the series  $\sum_{n=1}^{\infty} \frac{1}{n^2}$  converges or diverges.
33. Check whether  $\sum_{n=2}^{\infty} \frac{1+n \ln n}{n^2+5}$  converges or diverges.

(6 × 5 = 30 marks)

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**Part D (Essay Type)***Answer any two questions.*

34. The series  $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$  converges to  $\sin x$  for all  $x$ .
- (a) Find the first six terms of the series for  $\cos x$ . For what values of  $x$  should the series converge?
- (b) By replacing by  $2x$  in the series for  $\sin x$ , find a series that converges to  $\sin 2x$  for all  $x$ .
35. Find the Taylor series and Taylor polynomials generated by  $f(x) = \cos x$  at  $x = 0$ .
36. Find the length of the curve  $x = 8\cos t + 8t \sin t$ ,  $y = 8\sin t - 8t \cos t$ ,  $0 \leq t \leq \pi/2$ .

(2 × 10 = 20 marks)