

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2010
(CCSS)

Mathematics—Core Course
MM 3B 03—CALCULUS

Time : Three Hours

Maximum Weightage : 30

I. Objective Type questions
(Answer all questions, weightage 12 X ¼ = 3)

1. The average rate of change of the function $f(x) = x^3 + 1$ over the interval $[2, 3]$ is ...
2. If $2 - x^2 \leq g(x) \leq 2 \cos x$ for all x , then $\lim_{x \rightarrow 0} g(x) = \dots$
3. Suppose $f'(x) = 2x$ for all x , and if $f(0) = 0$ then $f(2) = \dots$
4. The critical points of f if $f'(x) = x(x - 1)$
5. The linearization of $f(x) = x^4$ at $x = 1$ is ...
6. $\lim_{x \rightarrow \infty} \frac{2x+3}{5x+7} = \dots$
7. A function with continuous first derivative is said to be ---
8. A differentiable function is always ...
9. $\frac{d}{dx} \int_1^{\sin x} 3t^2 dt = \dots$
10. The volume of a solid of known integrable cross-section area $A(x)$ from $x = a$ to $x = b$ is ...
11. The turning effect of a force about the origin is called ...
12. When a body moves a distance d along a straight line as a result of being acted on by a force of constant magnitude F in the direction of the motion, the work done by the force on the body is ...

II. Short answer type questions:
Answer all question (9X1 = 9 weightage)

13. Find $\lim_{x \rightarrow 5} \frac{x-5}{x^2-25}$
14. If $f(x) = x + 1$, find an open interval about 4 on which $|f(x) - 5| < 0.01$.
15. Find the function $f(x)$ whose derivative is $\sin x$ and whose graph passes through the point $(0, 2)$.
16. Find the interval on which the function $g(t) = -t^2 - 3t + 3$ is increasing

Turn over

17. What is the smallest perimeter possible for a rectangle whose area is 16 cm^2 ?
18. State the Mean Value Theorem for Definite Integrals
19. Evaluate $\int_0^3 \sqrt{y+1} \, dy$
20. Find the centre of mass of a wire of constant density δ shaped like a semicircle of radius 'a'
21. Find the work done by a variable force $F(x) = 1/x^2 \text{ N}$ along the x - axis from $x = 1 \text{ m}$ to $x = 10 \text{ m}$.

III. Short Essay or Paragraph Questions

Answer any 5 questions from 7 (5X2 = 10 weightage)

22. Show that $\lim_{x \rightarrow \infty} \frac{1}{x} = 0$
23. Using *Sandwich Theorem* find the asymptotes of the curve

$$y = 2 + \frac{\sin x}{x}$$
24. Show that the value of $\int_0^1 \sqrt{1 + \cos x} \, dx$ cannot possibly be 2.
25. Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the $x =$ axis and the line $y = x - 2$
26. Find the volume of the solid of revolution of the solid generated by revolving the region between the y -axis and the curve $x = 2/y$, $1 \leq y \leq 4$
27. Find the length of the plane curve $y = (x/2)^{2/3}$ from $x = 0$ to $x = 2$
28. Show that the centre of mass of a straight, thin strip or rod of constant density lies half way between its two ends.

IV. Essay Questions:

Answer any 2 questions (2X4 = 8 weightage)

29. Consider the function f defined by

$$f(x) = \begin{cases} x^2 - 1; & -1 \leq x < 0 \\ 2x; & 0 < x < 1 \\ 1; & x = 1 \\ -2x + 4; & 1 < x < 2 \\ 0; & 2 < x < 3 \end{cases}$$

- a) Find $f(-1)$
- b) Does $\lim_{x \rightarrow -1^+} f(x)$ exist

c) Does $\lim_{x \rightarrow -1^+} f(x) = f(-1)$

d) Is f continuous at $x = -1$

30. Define absolute extrema values of a function

Find the absolute extrema values of $g(t) = 8t^2 - t^4$ on $[-2, 1]$

31. The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the x -axis to generate a solid. Find the volume of the solid.