

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2012

Mathematics—Core Course

MM 3B 03—CALCULUS

: Three Hours

Maximum : 30 Weightage

Part I

I. Answer all the *twelve* questions :

- 1 Write the domain and range of the function $y = \sqrt{1 - x^2}$.
- 2 If $f(x) = x + 5$, $g(x) = x^2 - 3$ find $(g \circ f)(0)$.
- 3 Graph the function $f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ 2 - x, & 1 \leq x \leq 2 \end{cases}$
- 4 What real numbers satisfy the equation $\lfloor x \rfloor = 0$?
- 5 Find $\lim_{x \rightarrow 2^+} \left(\frac{x^2 - 3x + 2}{x^3 - 4x} \right)$
- 6 For what value of 'a' is $f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax, & x \geq 3 \end{cases}$ continuous for every x ?
- 7 Find the critical points of the function $f(x) = x^4 - 4x^3 + 10$.
- 8 Define linearization of $f(x)$ at $x = a$.
- 9 Express $1 - 2 + 4 - 8 + 16 - 32$ in sigma notation.
- 10 Find the average value of $f(x) = x^2 - 1$ is $[0, \sqrt{3}]$.
- 11 Find the work done by the force $f(x) = 30x$ along x axis from $x = 0$ m to $x = 2$ m.
- 12 Write the shell formula for revolution about y axis.

(12 × ¼ = 3 weightage)

Turn over

Part II

II. Answer all the *nine* questions :

13 Find the vertex and axis of the parabola $y = -\frac{1}{2}x^2 - x + 4$.

14 $f(x) = \begin{cases} 1-x^2, & x \neq 1 \\ 2, & x = 1 \end{cases}$. Does $\lim_{x \rightarrow 1} f(x)$ exist? Why?

15 At what points does the function $y = [x - 1] + \sin x$ is continuous?

16 Find the value of c in the mean value theorem for the function $f(x) = x + \frac{1}{x}$ in $\left[\frac{1}{2}, 2\right]$.

17 Use Sandwich theorem to find the asymptotes of the curve $y = 2 + \frac{\sin x}{x}$.

18 Evaluate $\int_{-1}^1 3x^2 \sqrt{x^3 + 1} dx$.

19 Find the area between $y = \sec^2 x$ and $y = \sin x$ from $x = 0$ to $x = \frac{\pi}{4}$.

20 Define moment of a system about origin.

21 Write the formula for finding centre of mass of a thin rod along x axis with density $\delta(x)$
(9 × 1 = 9 weight)

Part III

III. Answer any *five* questions from seven:

22 $f(x) = \frac{(x+3)[x+2]}{x+2}$, find $\lim_{x \rightarrow -2^+} f(x)$ and $\lim_{x \rightarrow -2^-} f(x)$.

23 Test the continuity of the function $f(x) = x \sin \frac{1}{x}$.

24 Find two positive integers whose sum is 20 and whose product is as large as possible.

- 25 Find the area of the region between and the graph of $f(x) = x^3 - x^2 - 2x$, $1 \leq x \leq 2$.
- 26 Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about line $x = 3$.
- 27 Find the length of the curve $y = \frac{4\sqrt{2}}{3}x^{3/2} - 1$ for $0 \leq x \leq 1$.
- 28 Find the centre of mass of a thin plate of constant density and covering the region bounded by the parabola $y = x - x^2$ and the line $y + x = 0$.

(5 × 2 = 10 weightage)

Part IV

Answer any two questions from three :—

29 $\lim_{x \rightarrow 5} \sqrt{x-1} = 2$. Find a $\delta > 0$ that works for $\epsilon = 1$.

30 Find the asymptotes of the curve :

$$f(x) = \frac{x^3 - 3}{2x - 4} \text{ and find the dominant terms.}$$

31 Find the area of the surface generated by revolving the curve $y = \frac{x^3}{9}$, $0 \leq x \leq 2$ about x axis.

(2 × 4 = 8 weightage)