

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2018
(CUCBCSS—UG)

Complementary Course

MAT 4C 64—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

Part A (Objective Type)

Answer all questions.

Each question carries 1 mark.

1. What do you mean by fundamental period ?
2. Write the characteristic equation of the differential equation $2y'' + 10y' + 25y = 0$.
3. Write the 2 dimensional wave equation.
4. Give an example of a function which is neither odd nor even.
5. State the existence and uniqueness theorem for initial value problem.
6. Find the Laplace transform of $f'(t)$.
7. Find the Wronskian of $e^{\lambda x}$ and $xe^{\lambda x}$.
8. Find $L^{-1}\left(\frac{1}{s/a - 1}\right)$.
9. Find $L(t + e^t)$.
10. State the second shifting theorem of Laplace transform.
11. Write the error estimate for Simpson's rule.
12. Write the formula for Runge-Kutta method.

(12 × 1 = 12)

Part B (Short Answer Type)

Answer any **nine** questions.

Each question carries 2 marks.

13. Find the Fourier cosine series of $f(x) = \pi - x, 0 < x < \pi$.
14. Show that $u = e^{-a^2 c^2 t} \sin wx$ is a solution of heat equation.
15. Apply Picard's iteration to solve $y' = y - x^2, y(0) = 1$. Also find $y(0, 1)$ and $y(0, 2)$.
16. By reducing to first order solve $y'' + (y')^2 = 0$.

17. Find fundamental set of solutions of $2t^2y'' + 3ty' - y = 0$ $t > 0$. Given that $y_1(t) = t^{-1}$ is a solution.
18. Solve $y'' + 8y' + 16y = 0$.
19. Find the inverse transform of $\frac{4}{(s+1)(s+2)}$.
20. Solve the initial value problem $y'' + y' - 6y = 1$ $y(0) = 0$, $y'(0) = 1$.
21. Find $L(te^{-t} \sin t)$.
22. Using convolution property find $L^{-1}\left[\frac{1}{s(s^2 + 4^2)}\right]$.
23. Use Trapezoidal rule with $n = 4$ to estimate $\int_1^2 x^2 dx$.
24. Solve the partial differential equation $u_{yy} + 4u = 0$.

(9 × 2 = 18 marks)

Part C (Short Essays)

Answer any **six** questions.
Each question carries 5 marks.

25. Solve $(x^2D^2 + 3xD + 1)y = 0$.
26. Find the general solution of the differential equation $y'' - 2y' + 5y = 5x^3 - 6x^2 + 6x$.
27. Find the Laplace transform of $e^{-2t}u(t-3)$.
28. Find $L^{-1}\left(\frac{se^{-2s}}{s^2 + \pi^2}\right)$.
29. Find the Fourier series expansion of:
- $$f(x) = \begin{cases} \sin x & 0 < x < \pi \\ 0 & \pi < x < 2\pi \end{cases}$$
30. Solve $xy'' - y' = (3+x)x^2e^x$.
1. Find the solution of $u_x + u_y = 0$ by separating variables.
2. Find approximate solution of $y' + y = e^x$ $y(0) = 0$.
3. Obtain the half-range cosine series of $f(x) = x$ when $0 < x < 2$.

(6 × 5 = 30 marks)

Part D

Answer any **two** questions.
Each question carries 10 marks.

34. Solve the non-homogeneous equation : $y'' - y' - 2y = 10 \cos x$.
35. Solve the initial value problem $y'' + 2y' + 2y = r(t)$

$$\text{where } r(t) = \begin{cases} 10 \sin 2t & \text{if } 0 < t < \pi \\ 0 & \text{if } t > \pi \end{cases}$$

$$y(0) = 1 \quad y'(0) = -5.$$

Using Laplace transforms.

36. Find the Fourier series of the functions

$$f(x) = \begin{cases} x + x^2 & -\lambda < x < \pi \\ \pi^2 & x = \pm \pi \end{cases}$$

Deduce that $1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$.

(2 × 10 = 20 marks)