

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2017

(CUCBCSS-UG)

Complementary Course

MAT 4C 04—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

Part A

Answer all questions.

Each question carries 1 mark.

1. Find the general solution of $y'' + \omega^2 y = 0$.
2. Find $(D^2 + 3D) \cosh 3x$.
3. Write the general form of the Cauchy-Euler equation.
4. Find $L(a + bt + ct^2)$.
5. $L(f') = \underline{\hspace{2cm}}$.

(a) $L(f) - Sf(0)$,	(b) $L(f) - f(0)$,
(c) $SL(f) - f(0)$,	(d) $SL(f) - f'(0)$.
6. $L^{-1}\left(\frac{1}{s^2 + a^2}\right) = \underline{\hspace{2cm}}$.
7. Write the second shifting theorem of Laplace transform.
8. Sketch $f(x) = |x|$ for $-\pi < x < \pi$.
9. Find a_0 in the Fourier series expansion of $f(x) = x^2, -\pi < x < \pi$.
10. Write the Picard's iteration formula to find a numerical solution of $y' = f(x, y), y(x_0) = y_0$.
11. Write the one-dimensional wave equation.
12. Find the period of $\cos \pi x$.

(12 × 1 = 12 marks)

Part B

Answer any nine questions.

Each question carries 2 marks.

13. Find the general solution of $(9D^2 + 6D + 1)y = 0$.
14. Solve $x^2 y'' - 3xy' + 4y = 0$.
15. Find the Laplace transform of $(t + 1)^2 e^t$.

Turn over

16. Find $L^{-1}\left(\frac{60 + 6s^2 + s^4}{s^7}\right)$.
17. If $f(t) = t$ and $g(t) = e^{at}$ find the convolution $(f * g)(t)$.
18. Find the Fourier cosine series of the function $f(x) = \pi - x$ in $0 < x < \pi$.
19. Prove that product of an even function and an odd function is an odd function.
20. Apply Picard's method to solve the initial value problem $y' = x^2 + y$, $y(0) = -1$.
21. Solve the partial differential equation $u_{xy} = u$.
22. Evaluate $\int_0^8 \frac{dx}{1+x^2}$ using Trapezoidal rule taking $h = 1$.
23. Solve the initial value problem $y'' + y' = 0$, $y(0) = 5$, $y'(0) = -3$ using Laplace transform.
24. Derive the Euler's formula to solve the differential equation $y' = f(x, y)$, $y(x_0) = y_0$.

(9 × 2 = 18 marks)

Part C

Answer any six questions.

Each question carries 5 marks.

25. Find the general solution of $y'' + 2y' + y = 2x + x^2$.
26. Find the general solution of $(D^2 + 3D - 4)y = 8\cos 2x$.
27. Solve $y'' + y = \sec x$ by the method of variation of parameters.
28. Find the inverse Laplace transform of $\frac{1}{(s + \sqrt{2})(s - \sqrt{3})}$.
29. Solve the initial value problem by Laplace transform $y' + 3y = 10\sin t$, $y(0) = 0$.
30. Find the Fourier series of the function $f(x) = x^2$, $-\pi < x < \pi$.
31. Using Laplace transform solve the integral equation $y(t) = 1 - \int_0^t (t - \tau) y(\tau) d\tau$.
32. Find the deflection $w(x, t)$ of a string of length $L = 2\pi$ when $c^2 = 1$, the initial velocity is zero and initial deflection is $0.1(\pi^2 - x^2)$.
33. Evaluate $\int_1^7 \frac{dx}{x}$ using Simpson's rule by dividing $[1, 7]$ into 6 equal parts.

(6 × 5 = 30 marks)

Part D

Answer any two questions.

Each question carries 10 marks.

34. Solve the initial value problem $y'' + 1.2y' + 0.36y = 4e^{-0.6x}$, $y(0) = 0$, $y'(0) = 1$.
35. Solve $y'' + 2y' + 5y = e^{-t} \sin t$, $y(0) = 0$. Using Laplace transform. Given $y'(0) = 1$.
36. Use improved Euler's method to determine $y(0.2)$ in two steps from $\frac{dy}{dx} = x^2 + y$, given that $y(0) = 1$.

(2 × 10 = 20 marks)