C 3964

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Name.......................................

Reg. No....................................

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2016

(CUCBCSS—UG)

Core Course—Mathematics

MAT AB 04—THEORY OF EQUATIONS, MATRICES AND VECTOR CALCULUS

Time : Three Hours

Maximum : 80 Marks

Part A (Objective Type)

Answer **all** *twelve questions.*

1. If α, β, γ are the roots of $2x^3 + 3x^2 - x - 1 = 0$ find the equation whose roots are $\alpha - 1, \beta - 1, \gamma - 1$.
2. State Descarte's rule of signs.
3. If α, β, γ are the roots of $x^3 - px^2 + qx - r = 0$ find the value of $\sum \alpha^2$.
4. If α, β, γ .... are the roots of $f(x) = 0$, write the equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$....
5. What is the rank of a unit matrix of order n ?
6. If $A = [a_{ij}]$ is an $m \times n$ matrix and $a_{ij} = 4$, for all i, j then rank of A is ————.
7. A system of m homogeneous linear equations $AX = 0$ in n unknowns has only trivial solution if ————.
8. For what value of a the system of equations $ax + y = 1, x + 2y = 3, 2x + 3y = 5$ are consistent.
9. If the number of variables in a non-homogeneous system $AX = B$ is n, then the system possesses a unique solution if ————.
10. Find the parametric equation of a line through the point $(-2, 0, 4)$ and parallel to the vector $2i + 4j - 2k$.
11. Find the unit vector tangent to the curve $r(t) = ti + (2/3)t^{3/2}\, k$.
12. Write the equations relating rectangular and spherical co-ordinates.

(12 × 1 = 12 marks)

Turn over

Part B (Short Answer Type)

Answer any **nine** *questions.*

13. Solve $8x^3 - 14x^2 + 7x - 1 = 0$ whose roots are in geometric progression.

14. Find the equation whose roots are the roots of $x^3 + 3x^2 - 2x - 4 = 0$ increased by 3.

15. If $\alpha, \beta, \gamma, \delta$ are the roots of $x^4 + px^3 + qx^2 + rx + s = 0$ evaluate $\sum \alpha^2 \beta$.

16. If $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ then rank of A^2 is :

17. Under what condition the rank of the matrix $A = \begin{bmatrix} 2 & 4 & 2 \\ 3 & 1 & 2 \\ 1 & 0 & x \end{bmatrix}$ is 3.

18. Show that the product of the characteristic roots of a square matrix of order n is equal to the determinant of the matrix.

19. If α is a characteristic root of a non-singular matrix A, then prove that $\frac{|A|}{\alpha}$ is a characteristic root of *adj* A.

20. Show that the characteristic roots of a Hermitian matrices are all real.

21. Find the velocity and acceleration vectors of $r(t) = e^t \boldsymbol{i} + \frac{2}{9} e^{2t} \boldsymbol{j}$ at $t = \ln 3$.

22. Find the equation for the cylinder $x^2 + (y-3)^2 = 9$ in cylindrical co-ordinates.

23. Evaluate $\int_0^1 \left((3t^2)\boldsymbol{i} + 2\boldsymbol{j} - (t-3)\boldsymbol{k} \right) dt$.

24. Fine the curvature of $r(t) = (a \cos t)\boldsymbol{i} + (a \sin t)\boldsymbol{j}$.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*

25. If α, β, γ are roots of $x^3 - x - 1 = 0$, find the equation whose roots are $\frac{1+\alpha}{1-\alpha}, \frac{1+\beta}{1-\beta}, \frac{1+\gamma}{1-\gamma}$. Hence write down the values of $\sum\left(\frac{1+\alpha}{1-\alpha}\right)$.

26. If α, β, γ are roots of $x^2 + qx + r = 0$, find the equation whose roots are $(\beta - \gamma)^2, (\gamma - \alpha)^2, (\alpha - \beta)^2$.

27. Solve the equation $x^2 - 12x - 65 = 0$ by Cardan's method.

28. For the matrix A, find non-singular matrices P and Q such that PAQ is in normal form, where

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}.$$

29. Prove that rank of a non-singular matrix is equal to the rank of its reciprocal matrix.

30. Using matrix method solve the equations :

$$x + 2y + 3z = 14$$
$$3x + y + 2z = 11$$
$$2x + 3y + z = 11.$$

31. Find the point where the line $x = \frac{8}{3} + 2t, y = -2t, z = 1 + t$ intersects the plane $3x + 2y + 6z = 6$.

32. Find the distance from the point $S(1, 1, 5)$ to the line $L : x = 1 + t, y = 3 - t, z = 2t$.

33. Find the latent roots and latent vectors of the matrix $A = \begin{bmatrix} a & h & g \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$.

(6 × 5 = 30 marks)

Part D (Essay Type)

Answer any **two** *questions.*

34. Solve the equation $x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0$.

35. Find the characteristic equation of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 0 & -3 & 0 \\ 1 & 1 & -1 \end{bmatrix}$ and verify that it is satisfied by A and hence obtain A^{-1}.

36. Find the binormal vector and torsion for the space curve

$$r(t) = (a\cos t)\,\boldsymbol{i} + (a\sin t)\,\boldsymbol{j} + bt\boldsymbol{k},\ a, b \geq 0,\ a^2 + b^2 = 1.$$

(2 × 10 = 20 marks)