

**FOURTH SEMESTER B.Sc. DEGREE (SUPPLEMENTARY/IMPROVEMENT)
EXAMINATION, MAY 2016**

(UG—CCSS)

Complementary Course

MM 4C 04—MATHEMATICS

Time : Three Hours

Maximum : 30 Weightage

Section A*Answer all questions.**Each question carries a weightage of $\frac{1}{4}$.*

1. Is the equation $y'' = \sqrt{y'^2 + 1}$ linear or non-linear ?
2. Is $y = e^{-x}$ a solution of $y'' + y = 0$?
3. Solve $y'' + 7y = 0$.
4. Find the Laplace Transform of $f(t) = t^9$.
5. Define the unit impulse function.
6. Find the inverse Laplace Transform of $F(s) = \frac{2}{2s - 3}$.
7. Find the fundamental period of $\cos 2\pi x$.
8. Is the following function even or odd or neither $x^2 \cos nx$.
9. What is the 2 dimensional Laplace equation ?
10. Define the Lipschitz condition.
11. What is an initial value problem ?
12. State Simpson's rule.

(12 × $\frac{1}{4}$ = 3 weightage)**Section B***Answer all questions.**Each question carries a weightage of 1.*

13. Find the Wronskian of the functions $y_1 = x^2$ and $y_2 = x^2 \ln x$.
14. Find the solution of $y'' + 4y' + 4y = 0$.
15. Find the Laplace Transform of $f(t) = \cosh 7t$.

Turn over

16. Find the inverse Laplace Transform of $F(s) = (s-2)^{-5}$.
17. Is $u = \sin ct \sin x$ a solution of the wave equation (with suitable c) ?
18. Solve $u_{xy} = u_x$.
19. Solve $u_y = u$.
20. Show that $f(x, y) = |\sin y| + x$ satisfies the Lipschitz condition with $m = 1$.
21. Apply Euler's method and compute y_1, y_2, \dots, y_5 with $h = 0.02$, given $y' = \frac{(y-x)}{(y+x)}, y(0) = 1$.
(9 × 1 = 9 weightage)

Section C

Answer any five questions.
Each question carries a weightage of 2.

22. Solve $y'' + y' = 2 + 2x + x^2, y(0) = 8, y'(0) = -1$.
23. Solve $x^2 y'' + xy' + y = 0$.
24. Find the Laplace Transform of $F(t) = te^{-2t} \sin 2t$.
25. State the convolution theorem and use it to evaluate the inverse $h(t)$ of $H(s) = s(s^2 + a^2)^{-2}$.
26. Find the Fourier sine series of $f(x) = \pi - x, 0 < x < \pi$.
27. Using Runge Kutta Method, find y when $x = 0.2$, given $y' = \frac{y^2 - x^2}{y^2 + x^2}, y(0) = 1$.
28. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Trapezoidal rule, taking $h = 0.25$.

Section D

(5 × 2 = 10 weightage)

Answer any two questions.
Each question carries a weightage of 4.

29. Solve $y'' + y = \sec x$.
30. Solve by the method of Laplace Transforms: $y'' + y = t, y(0) = 1, y'(0) = -2$.
31. Find the Fourier series expansion of $f(x) = \frac{x^2}{2}, -\pi < x < \pi$. Hence show that

$$1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots = \frac{\pi^2}{6}$$

(2 × 4 = 8 weightage)