C 5129

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Name.....

Reg. No.....

FOURTH SEMESTER B.Sc. DEGREE (SUPPLEMENTARY/IMPROVEMENT) EXAMINATION, MAY 2016

(UG–CCSS)

Core Course—Mathematics

MM 4B 04—CALCULUS AND ANALYTIC GEOMETRY

Time : Three Hours

Maximum : 30 Weightage

I. Answer *all* questions.

1 Find the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

2 Evaluate $\int_0^{\pi} \frac{\sin t}{2 - \cos t}\, dt$.

3 Evaluate $\frac{d}{dt}\left(\tan h \sqrt{1+t^2}\right)$.

4 Write the parametric equations of the circle $x^2 + y^2 = 1$.

5 Find the vertices of the hyperbola $\frac{y^2}{4} - \frac{x^2}{5} = 1$.

6 Examine whether $3x^2 - 6xy + 3y^2 + 2x - 7 = 0$ represents a parabola, ellipse or hyperbola.

7 Define absolute convergence.

8 Show that x^2 grows faster than $ln\, x$ as $x \to \infty$.

9 Find y if $ln\, y = 3t + 5$.

10 Evaluate $\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x}$.

11 Find the Taylor polynomial of order 1 generated by $f(x)\, ln\, x$ at $a = 1$.

12 Examine whether $\sum_{n=1}^{\infty} n^2$ converges or diverges.

(12 × ¼ = 3 weightage)

Turn over

II. Answer *all* questions.

13 Examine whether the series

$$5+\frac{2}{3}+1+\frac{1}{7}+\frac{1}{2}+\frac{1}{3!}+\frac{1}{4!}+\ldots.+\frac{1}{k!}+\ldots\cdot\text{converges.}$$

14 Prove that the alternating series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots.$ converges.

15 Find $\frac{dy}{dx}$ if $y=x^x, x>0$.

16 Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{4\cos\theta}{3+2\sin\theta}\,d\theta$.

17 Graph the set of points whose polar co-ordinates satisfy the conditions $r\leq 0$ and $r\leq 0\,\theta=\frac{\pi}{4}$.

18 Evaluate $\frac{d}{dx} ln_{10}(3x+1)$.

19 Evaluate $\lim_{x\to 0}\frac{x-\sin x}{x^3}$.

20 Find the Maclausin's series for $f(x)=\frac{1}{1+x}$.

21 For what values of x do the series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}$ converges.

(9 × 1 = 9 weightage)

III. Answer any *five* questions.

22 Find the Maclaurin's series for $f(x)=\sin 3x$.

23 Find the centroid of the first quadrant of the astroid $x=\cos^3 t, y=\sin^3 t,\ 0\leq t\leq 2\pi$.

24 Evaluate $\int_0^1 \sin h^2 x\,dx$.

25 Find the Taylor polynomial generated by $f(x)=\cos x$ at $x=0$.

26 Evaluate $\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{x}\right)$.

27 Graph the curve $r^2=4\cos q$.

28 The co-ordinate axes are to be rotated through an angle a to produce an equation for the curve $2x^2 + \sqrt{3}\,xy + y^2 - 10 = 0$ and has no cross production. Find a and the new equation. Identify the curve.

(5 × 2 = 10 weightage)

IV. Answer any *two* questions.

29 Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.

30 Find the area of the region in the plane enclosed by the cardioid $r = 2\,(1 + \cos\theta)$.

31 Find the length of the cardioid $r = 1 - \cos\theta$.

(2 × 4 = 8 weightage)