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Name..................................

Reg. No..................................

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL/MAY 2015

(U.G.—CCSS)

Core Course—Mathematics

MM 4B 04—CALCULUS AND ANALYTIC GEOMETRY

Time : Three Hours

Maximum : 30 Weightage

I. Answer *all* questions :—

1 Find y if $ln\, y = 3t + 5$.

2 Evaluate $\int_0^{\pi/2} e^{\sin x} \cos x \, dx$.

3 Evaluate $\frac{d}{dx} ln_{10}(3x+1)$.

4 Evaluate $\lim_{x \to 0} \frac{\sqrt{1+x}-1}{x}$.

5 Integrate coth $5x$.

6 Examine whether $\sum_{n=1}^{\infty} \frac{n+1}{n}$ converges or diverges.

7 Define an alternating series.

8 Find the Taylor series for $f(x) = e^x$ at $x = 0$.

9 Find the Taylor polynomial of order 0 generated by $f(x) = \frac{1}{x}$, $a = 2$.

10 Find the focus of the parabola $y^2 = 10x$.

11 Find the eccentricity of the hyperbola $9x^2 - 16y^2 = 144$.

12 Write the parametric equation of the circle $x^2 + y^2 = a^2$.

($12 \times ¼ = 3$ weightage

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II. Answer all *nine* questions :—

13 Evaluate $\int_{-\pi/2}^{\pi/2} \frac{4\cos\theta}{3+2\sin\theta}\,d\theta$.

14 Integrate $2^{\sin x}\cos x$ with respect to x.

15 Evaluate $\lim_{x\to 0}\frac{x-\sin x}{x^3}$.

16 Show that $ln\ x$ grows slower than x as $x \to \infty$.

17 Examine whether $x^2 + xy + y^2 - 1 = 0$ represents a parabola, ellipse or hyperbola.

18 Find the vertices of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

19 Find the Taylor series for $f(x) = \cos x$ at $x = 0$.

20 For what values of x do the series $\sum_{n=1}^{\infty} (-1)^{n-1}\frac{x^n}{n}$ converges.

21 Examine whether the series

$$5 + \frac{2}{3} + 1 + \frac{1}{7} + \frac{1}{2} + \frac{1}{3!} + \frac{1}{4!} + \ldots\ldots + \frac{1}{k!} + \ldots\ldots$$ converges.

($9 \times 1 = 9$ weightage)

III. Answer any *five* questions :—

22 Find the directrix of the parabola $r = \frac{25}{10 + 10\cos\theta}$.

23 Graph the curve $r^2 = 4\cos\theta$.

24 Find the tangent to the right-hand hyperbola branch $x = \sec t,\ y = \tan t,\ -\pi/2 < t < \frac{\pi}{2}$ at the point $(\sqrt{2}, 1)$ where $t = \frac{\pi}{4}$.

25 Show that the Maclaurin's series for $\sin x$ converges to $\sin x$ for all x.

26 Investigate the converges of the series $\sum_{n=1}^{\infty} \frac{(2n)!}{n!\,n!}$.

27 Evaluate $\int_0^1 \sinh^2 x\, dx$.

28 Show that $\lim_{x \to 0^+} (1+x)^{\frac{1}{x}} = e$.

(5 × 2 = 10 weightage)

Answer any *two* questions :

29 Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.

30 Multiply the geometric series $\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \dots + x^n + \dots \frac{1}{1-x}$, for $|x| < 1$, by itself to get a power series for $\frac{1}{(1-x)^2}$, for $|x| < 1$.

31 Find the length of the astroid $x = \cos^3 t,\; y = \sin^3 t,\; 0 \le t \le 2\pi$.

(2 × 4 = 8 weightage)