C 61216

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Name..............................

Reg. No..............................

FOURTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, APRIL 2019

(CUCBCSS—UG)

Common Course for LRP

MA T4 B04—THEORY OF EQUATION, MATRICES AND VECTOR CALCULUS

Time : Three Hours

Maximum : 80 Marks

Part A (Objective Type)

Answer all **twelve** *questions.*
Each question carries 1 mark.

1. If α, β, γ are the roots of $x^3 - px^2 + qx - r = 0$ find the value of $\sum \alpha^2$.
2. Define a reciprocal equation.
3. State Descarte's rule of signs.
4. If α, β, γ are the roots of $f(x) = 0$, write the equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$.
5. Rank of the matrix $A = \begin{bmatrix} 0 & 2 & 3 \\ 0 & 4 & 6 \\ 0 & 6 & 9 \end{bmatrix}$ is .... .
6. If A is a non-zero column matrix and B is a non-zero row matrix then rank (AB) is .... .
7. The system $AX = 0$ in n unknowns has a non-trivial solution if ———.
8. For what value of a the system of equations $ax + y = 1, x + 2y = 3, 2x + 3y = 5$ are consistent.
9. If A is an n-rowed non-singular matrix, X and B are $n \times 1$ matrices, then the system of equations $AX = B$ has ——— solution.
10. Find the parametric equation of a line through the point $(3, -4, -1)$ and parallel to the vector $\mathbf{i} + \mathbf{j} + \mathbf{k}$.

Turn over

11. Find the unit vector tangent to the curve $r(t) = (2\cos t)\,\mathbf{i} + (2\sin t)\,\mathbf{j} + \sqrt{5}t\,\mathbf{k},\ 0 \le t \le \pi$.

12. Write the equations relating rectangular and cylindrical co-ordinates.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 marks.

13. Solve $x^3 - 12x^2 + 39x^2 - 28 = 0$ whose roots are in arithmetic progression.

14. Transform $x^3 - 6x^2 + 5x + 12 = 0$ into an equation lacking the second term.

15. If $\alpha, \beta, \gamma, \delta$ are the roots of $x^4 + px^3 + qx^2 + rx + s = 0$ evaluate $\sum \alpha^2\beta\gamma$.

16. If $A = \begin{bmatrix} 1 & 5 & 2 \\ 3 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 1 & 4 \\ 2 & 1 \end{bmatrix}$, then rank of AB is :

17. Under what condition the rank of the matrix $A = \begin{bmatrix} 2 & 4 & 2 \\ 3 & 1 & 2 \\ 1 & 0 & x \end{bmatrix}$ is 3.

18. Show that corresponding to a characteristic vector X of a square matrix A, there exist one and only one characteristic root.

19. If A is non-singular, prove that the eigen values of A^{-1} are the reciprocals of the eigen values of A.

20. Show that the characteristic roots of a Hermitian matrices are all real.

21. Find the velocity and acceleration vectors of $r(t) = (t+1)\,\mathbf{i} + (t^2 - 1)\,\mathbf{j}$ at $t = 1$.

22. Find a spherical co-ordinate equation for the sphere $x^2 + y^2 + (z-1)^2 = 1$.

23. Evaluate $\int_0^{\pi} ((\cos t)\,\mathbf{i} + \mathbf{j} - (2t)\,\mathbf{k})\,dt$.

24. Find the curvature of $r(t) = t\,\mathbf{i} + (\ln \cos t)\,\mathbf{j},\ -\pi/2 < t < \pi/2$.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*
Each question carries 5 marks.

25. If α, β, γ are roots of $x^3 - x - 1 = 0$, find the equation whose roots are $\frac{1+\alpha}{1-\alpha}, \frac{1+\beta}{1-\beta}, \frac{1+\gamma}{1-\gamma}$. Hence write down the values of $\Sigma\left(\frac{1+\alpha}{1-\alpha}\right)$.

26. If α, β, γ are roots of $x^3 + qx + r = 0$, find the equation whose roots are $(\beta - \gamma)^2, (\gamma - \alpha)^2, (\alpha - \beta)^2$.

27. Solve the equation $x^4 + 6x^3 - 5x^2 + 6x + 1 = 0$.

28. Reduce the matrix A to its normal form and hence find the rank of A where :

$$A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}.$$

29. Show that rank (AA') = rank (A).

30. Find the latent roots and latent vectors of the matrix $A = \begin{bmatrix} a & h & g \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$.

31. Find the point where the line $x = \frac{8}{3} + 2t,\ y = -2t,\ z = 1 + t$ intersects the plane $3x + 2y + 6z = 6$

32. Find the distance from the point S (1, 1, 5) to the line L : $x = 1 + t, y = 3 - t, z = 2t$.

33. Using matrix method solve the equations :

$$x + y + z = 6$$

$$x - y - z = 2$$

$$2x + y - z = 1.$$

$(6 \times 5 = 30$ marks)

Part D (Essay Type)

Answer any **two** *questions.*
Each question carries 10 marks.

34. Solve the equation $x^3 - 3x^2 + 12x + 16 = 0$ by Cardan's method.

35. Show that the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ satisfies Cayley-Hamilton theorem. Hence obtain A^{-1}.

36. Find the binormal vector and torsion for the space curve $r(t) = (3 \sin t)\, \mathbf{i} + (3 \cos t)\, \mathbf{j} + 4t\mathbf{k}$.

(2 × 10 = 20 marks)