

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2011

(CCSS)

Core Course

Mathematics

MM 4B 04—CALCULUS AND ANALYTIC GEOMETRY

Time : Three Hours

Maximum : 30 Weightage

I. Answer all questions :

- 1 Find y if $\ln y = 3t + 5$.
- 2 Find $\frac{d}{dx} \log_{10}(3x + 1)$.
- 3 Evaluate $\lim_{x \rightarrow \left(\frac{\pi}{2}\right)^-} \frac{\sec x}{1 + \tan x}$.
- 4 Find $\lim_{n \rightarrow \infty} \frac{2^n}{5^n}$.
- 5 Give an example of a divergent sequence.
- 6 When does a sequence of real numbers $\{a_n\}$ converges to the real number 1 ?
- 7 Find a formula for the n^{th} term of the sequence 1, -1, 1, -1, ...
- 8 Examine the convergence of $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$.
- 9 Graph the set of points whose polar co-ordinates satisfy the condition $1 \leq r \leq 2$ and $0 \leq \theta \leq \frac{\pi}{2}$.
- 10 Show that the point $\left(2, \frac{\pi}{2}\right)$ lies on the curve $r = 2 \cos 2\theta$.

11 Find $\frac{dy}{dx}$ if $y = x^x$, $x > 0$.

12 Evaluate $\int 2^x dx$.

(12 × ¼ = 3 weightage)

II. Answer all nine questions :

13 Integrate $\frac{\log_2 x}{x}$ with respect to x .

14 Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1 - \left(\frac{x}{2}\right)}{x^2}$.

15 Show that $\sqrt{x^2 + 5}$ and $(2\sqrt{x} - 1)^2$ grow at the same rate as $x \rightarrow \infty$.

16 Examine the convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n 5}{4^n}$.

17 For what values of x does the power $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{2n-1}}{2n-1}$ converges.

18 Find the Maclourin's series for $\frac{1}{1+x}$.

19 State Rearrangement Theorem for absolute convergent series.

20 Write the standard polar equation of a ellipse with eccentricity e and semimajor axis a .

21 Show that $\sinh 2x = 2\sinh x \cosh x$.

(9 × 1 = 9 weightage)

III. Answer any five questions :

22 Solve the initial value problem $e^y \frac{dy}{dx} = 2x$, $x > \sqrt{3}$, $y(2) = 0$.

23 Show that $\lim_{x \rightarrow 0^+} (1+x)^{1/x} = e$.

24 Does the sequence whose n^{th} term is $a_n = \left(\frac{n+1}{n-1}\right)^n$ converge? If so, find $\lim_{n \rightarrow \infty} a_n$.

25 Prove that $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$.

26 Find a polar equation for the circle $x^2 + (y-3)^2 = 9$.

27 Show that the Taylor series generated by $f(x) = e^x$ at $x = 0$ converges to $f(x)$ for every real value of x .

28 Calculate e with an error of less than 10^{-6} .

(5 × 2 = 10 weightage)

IV. Answer any *two* questions :

29 Find the sum of series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.

30 Multiply the geometric series $\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \dots + x^n + \dots = \frac{1}{1-x}$, for $|x| < 1$, by itself

to get a power series for $\frac{1}{(1-x)^2}$ for $|x| < 1$.

31 Find the length of the astroid $x = \cos^3 t$, $y = \sin^3 t$, $0 \leq t \leq 2\pi$.

(2 × 4 = 8 weightage)