

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2012

(CCSS)

Mathematics

MM 4B 04—CALCULUS AND ANALYTIC GEOMETRY

: Three Hours

Maximum : 30 Weightage

I. Objective Type Questions (Answer all questions) :

- 1 Evaluate $\lim_{x \rightarrow 0^+} x \cot x$.
- 2 Show that $\ln x$ grows slower than x as $x \rightarrow \infty$.
- 3 Evaluate $\int_0^{\ln 2} 4e^x \sinh x \, dx$.
- 4 Find $\lim_{n \rightarrow \infty} \frac{\ln(n^2)}{n}$.
- 5 Define the convergence of a sequence.
- 6 Examine the convergence of $\sum_{n=1}^{\infty} \frac{2^n}{n^2}$.
- 7 Define Absolute convergence.
- 8 Find a formula for the n^{th} term of the sequence 1, -4, 9, -16, 25, ...
- 9 Find the radius of the circle $r = 6 \sin \theta$.
- 10 Graph the set $r \leq 0$ and $\theta = \frac{\pi}{4}$.
- 11 Find the polar equation of the hyperbola with eccentricity $\frac{3}{2}$ and directrix $x = 2$.
- 12 Find a polar equation for the circle $x^2 + (y - 3)^2 = 9$.

(12 × ¼ = 3 weightage)

Turn over

II. Short answer type questions (Answer *all nine* questions) :

13 Evaluate $\lim_{x \rightarrow \infty} \frac{\ln x}{2\sqrt{x}}$.

14 Evaluate $\int_0^1 \frac{2 dx}{\sqrt{3+4x^2}}$.

15 Show that $\sum_{n=1}^{\infty} n^2$ diverges.

16 Does the series $5 + \frac{2}{3} + 1 + \frac{1}{7} + \frac{1}{2} + \frac{1}{3!} + \frac{1}{4!} + \dots + \frac{1}{k!} + \dots$ converge?

17 For what values of x does the power series $\sum_{n=0}^{\infty} n! x^n$ converge?

18 Examine the convergence of $\sum_{n=0}^{\infty} \frac{2^n + 5}{3^n}$.

19 Show that the series $\frac{1}{1} + \frac{1}{3} + \frac{1}{7} + \frac{1}{15} + \dots$ converges.

20 Sketch the curve $r = 1 + \cos \theta$.

21 Find the slope of $r = -1 + \cos \theta$ at $\theta = \pm \frac{\pi}{2}$.

(9 × 1 = 9)

III. Short essay questions (Answer any *five* questions) :

22 Find $\lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$.

23 Evaluate $\lim_{x \rightarrow 0^+} x \cot x$.

24 Find the area of the region in the plane enclosed by the cardioid $r = 2(1 + \cos \theta)$.

25 Find the tangent to the right-hand hyperbola branch $x = \sec t$, $y = \tan t$, $-\frac{\pi}{2} < t < \frac{\pi}{2}$, at the point $(\sqrt{2}, 1)$ where $t = \frac{\pi}{4}$.

26 Sum the series $\sum_{n=1}^{\infty} \frac{3^{n-1} - 1}{6^{n-1}}$.

27 Find the Taylor series generated by $f(x) = \frac{1}{x}$ at $a = 2$.

28 Examine the convergence of the series $\sum_{n=2}^{\infty} \frac{1 + n \ln n}{n^2 + 5}$.

(5 × 2 = 10 weightage)

Essay questions (Answer any two questions):

29 The coordinate axes are to be rotated through an angle α to produce an equation for the curve $2x^2 + \sqrt{3}xy + y^2 - 10 = 0$ that has no cross product term. Find α and the new equation.

30 Does the series $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots + \frac{1}{n^2} + \dots$ converge?

31 Find the Taylor series and Taylor polynomial generated by $f(x) = \cos x$ at $x = 0$.

(2 × 4 = 8 weightage)