C 41424 (Pages : 3) Name.....

Reg. No.....

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2013

(CCSS)

Mathematics

MM4 B04—CALCULUS AND ANALYTIC GEOMETRY

Time : Three Hours Maximum : 30 Weightage

I. Answer *all* questions :

1 Evaluate $\int_0^{\pi/6} \tan 2x \ dx$.

2 Prove that $e^{x+\ln_2} = 2e^x$.

3 Evaluate $\lim_{x \to 0} \frac{3x - \sin x}{x}$.

4 Write the parametric equation of the circle $x^2 + y^2 = 1$.

5 Find the foci of the hyperbola $\frac{y^2}{4} - \frac{x^2}{5} = 1$.

6 Examine whether $3x^2 - 6xy + 3y^2 + 2x - 7 = 0$ represents a parabola, ellipse or hyperbola.

7 Find the directrix of the parabola $y^2 = 10x$.

8 Find the Taylor polynomial of order 1 generated by $f(x) = \ln x$ at $a = 1$.

9 Define Absolute convergence.

10 Examine whether $\sum_{n=1}^{\infty} n^2$ converges or diverges.

11 Evaluate $\frac{d}{dt}\left(\tan h \sqrt{1 + t^2}\right)$.

12 Show that x^2 grows faster than $\ln x$ as $x \to \infty$.

(12 × ¼ = 3 weightage)

Turn over

II. Answer all *nine* questions :

13 Find k if $e^{2k} = 10$.

14 Evaluate $\frac{d}{dx} \log_{10} (3x+1)$.

15 Find the sum of the series $\sum_{n=1}^{\infty} \frac{3^{n-1}-1}{6^{n-1}}$.

16 Fidn the Taylor series for $f(x) = e^x$ at $x = 0$.

17 Find the Maclaurin's series for $x \sin x$.

18 Find the eccentricity of the hyperbola $9x^2 - 16y^2 = 144$.

19 Graph the set of points whose co-ordinates satisfy the conditions $1 \le r \le 2$ and $0 \le \theta \le \frac{\pi}{2}$.

20 Find an equation for the hyperbola with eccentricity $\frac{3}{2}$ and directrix $x = 2$.

21 Find the Maclaurin's series for $f(x) = e^{-x}$.

(9 × 1 = 9 weightage)

III. Answer any *five* questions :

22 Evaluate $\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$.

23 Prove that if $\sum_{n=1}^{\infty} |a_n|$ converges then $\sum_{n=1}^{\infty} a_n$ converges.

24 Find the Taylor polynomial generated by $f(x) = \cos x$ at $x = 0$.

25 Find the Cartesian equation for the hyperbola centered at the origin that has a focus at (3, 0) and the line $x = 1$ as the corresponding directrix.

26 Does the sequence whose n^{th} term is $a_n = \left(\frac{n+1}{n-1} \right)^n$ converge ? If so find $\lim_{n \to \infty} a_n$.

27 Using Integral test examine whether the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.

28 Find the length of the cardioid $r = 1 - \cos\theta$.

(5 × 2 = 10 weightage)

IV. Answer any *two* questions :

29 Find the area inside the smaller loop of $r = 2\cos\theta + 1$.

30 Show that the Taylor series generated by $f(x) = e^x$ at $x = 0$ converges to $f(x)$ for every real value of x.

31 Solve the initial value problem :

$$e^y \frac{dy}{dx} = 2x,\ x > \sqrt{3},\ y(2) = 0.$$

(2 × 4 = 8 weightage)