FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2013

MM 4C 04—MATHEMATICS

Time : Three Hours   Maximum : 30 Weightage

Unit I

*(Answer all **twelve** questions.)*

1. Which of the following is not a solution of $y'' + y = 1$?

   (a) $1 + \cos x$.

   (b) $1 + \sin x$.

   (c) $2(1 + \cos x)$.

   (d) $1 + \cos x + \sin x$.

2. Find the general solution of $y'' - 8y = 0$.

3. Auxillary equation corresponding to $x^2 y'' - 2.5xy' - 2y = 0$:

   (a) $m^2 - 2.5m - 2 = 0$.

   (b) $m^2 - 3.5m - 2 = 0$.

   (c) $m^2 - 1.5m - 2 = 0$.

   (d) $m^2 + 3.5m - 2 = 0$.

4. If $y_1 = \sin 2x$ and $y_2 = \cos 2x$. Find $w(y_1, y_2)$.

5. Find the Laplace transform of $2t + 6$.

6. Find $L^{-1}\left[\dfrac{5s}{s^2 - 25}\right]$.

7. Define the unit step function $u(t - a)$.

8. $f(x) = x \cos nx$ is :

   (a) An even function.

   (b) Odd function.

   (c) Neither even nor odd.

   (d) Either even or odd.

9. Write the general form of the two dimensional Poisson's equation.

10. Solve the partial differential equation $u_{xy} + u_x = 0$.

11. Find the smallest positive period p of $\sin 2x$.

12. Sketch the function $f(x) = |x|$ for $-\pi < x < \pi$.

$(12 \times \frac{1}{4} = 3$ weightage$)$

Turn over

Unit II

*(Answer any **nine** questions.)*

13. Apply $(D+1)(D-2)$ to xe^{-x}.

14. Find the general solution of $(D^2 - D - 2)\, y = 0$.

15. Find two linear independent solutions of $x^2 y'' - 3xy' + 4y = 0$.

16. Verify that $y_p = 2x^2 - 6x + 7$ is a solution of $y'' + 3y' + 2y = 4x^2$.

17. Reduce to first order and solve $y'' = y'$.

18. Find the Laplace transform of $5\, e^{2t} \sinh 2t$.

19. If $L(f(t)) = F(s)$ prove that $L\left[e^{at} f(t)\right] = F(s-a)$.

20. Find $L^{-1}\left[\dfrac{s^2+1}{(s-1)^2}\right]$.

21. Show that $u = 2xy$ satisfies the Laplace's equation.

22. Find the solution of $u_x - u_y = 0$ by separating the variables.

23. Find a_0 in the Fourier series expansion of $f(x) = \begin{cases} -2x, & -\pi < x < 0 \\ 2x, & 0 < x < \pi, \end{cases}$ $p = 2\pi$.

24. Find the first two approximate solutions $y_1(x)$ and $y_2(x)$ of the initial value problem $y' = x + y,\ y(0) = 0$.

$(9 \times 1 = 9$ weightage$)$

Unit III

*(Answer any **five** questions.)*

25. Solve the initial value problem $y'' - 16y' + 13y = 4\, e^{3x}$, $y(0) = 2$, $y'(0) = 4$.

26. Using method of variation of parameters solve $y'' + 9y = \csc 3x$.

27. Find $e^t * e^{-t}$ where $*$ denotes the convolution.

28. Using method of partial fractions find $L^{-1}\left[\dfrac{-s-10}{s^2-s-2}\right]$.

29. Find $L^{-1}\left[\dfrac{w}{s^2(s^2+w^2)}\right]$ by convolution.

30. Solve the integral equation $y(t)=1+\displaystyle\int_0^t y(\tau)\,d\tau$.

31. Find the Fourier series expansion of $f(x)=2x,\ -1<x<1,\ p=2$.

32. Use Trapezoidal rule with $n=4$ to estimate $\displaystyle\int_1^3 (2x-1)\,dx$.

$(5 \times 2 = 10$ weightage$)$

Unit IV

(*Answer any* **two** *questions.*)

33. Apply Euler's method to solve $y'=x+y,\ y(0)=0,\ h=0.2$.

34. Find the Fourier series of $f(x)=x^2\ -\pi<x<\pi$.

35. Solve using Laplace transform $y'+10y=10\sin t,\ y(0)=0$.

$(2 \times 4 = 8$ weightage$)$