21175 (Pages : 3) Name..................................

Reg. No..................................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2011

(CCSS)

Core Course—Mathematics

MM 5B 08—DIFFERENTIAL EQUATIONS

: Three Hours Maximum Weight : 30

I. Answer *all* questions :

1. Does the differential equation $\frac{dy}{dt} = y$ have a solution passing through the point $(1, 0)$?
2. Is the differential equation $\frac{dy}{dt} + ty^2 = 0$ linear or non linear ?
3. Give an integrating factor of the equation : $ydx - xdy = 0$.
4. The differential equation $M(x, y) + N(x, y)\, y' = 0$ is exact if and only if ———.
5. Give the general solution of $\frac{d^2y}{dt^2} + b\frac{dy}{dt} + cy = 0$ whose characteristic equation has a root $\lambda + i\mu$.
6. Write a differential equation whose general solution is $c_1e^t + c_2te^t$.
7. Are the functions $\cos t$ and $\sin t$ linearly independent ?
8. The Laplace Transform of the function $t^2 + e^{2t}$ is ———.
9. If $h(t) = u_1(t) - u_2(t)$ where u_1 and u_2 are unit step functions, then $h(t) =$ ——— if $1 \le t < 2$.
10. If $F(s) = \mathcal{L}(f(t))$ exists for $s > a \ge 0$ and c is a constant, then $\mathcal{L}(e^{a}f(t)) =$ ——— $s > a + c$.
11. The fundamental period of the function $\cos(2\pi x)$ is ———.
12. State whether the function $f(x) = x.\cos x$ is even or odd.

$(12 \times \frac{1}{4} = 3)$

Turn over

II. Answer *all* questions.

13. Solve the initial value problem : $y' = y^{\frac{1}{3}}$; $y(0) = 0$ and give an interval in which the s
exists.

14. Prove that $\mu(x, y) = x$ is an integrating factor of the differential eq
$(3xy + y^2) + (x^2 + xy)y' = 0$.

15. Obtain the general solution of the equation $16y'' - 8y' + 145y = 0$.

16. Prove that $t^{1/2}$ and t^{-1} form a fundamental set of solutions of the equation $2t^2y'' + 3ty' -$

17. Prove that $f * g = g * f$, where * denotes the convolution product.

18. Find the Laplace Transform of the unit step function $u_c(t)$.

19. Find the Inverse Laplace Transform of the function $\frac{s^2}{s^4 - 1}$.

20. If f and g are periodic functions with same period T, show that any linear combination o
g is also T-periodic.

21. Let $f(x) = x$ where $0 \le x \le 1$. Find the 2-periodic even extension of f.

(9 × 1

III. Answer any *five* questions.

22. Solve the initial value problem $2dx + ye^{-x}dy = 0; y(0) = 0$.

23. Solve the initial value problem $y'' - y' + 0.25y = 0$; $y(0) = 2$ and $y'(0) = \frac{1}{3}$.

24. If y_1 and y_2 are solutions of $y'' + p(t)y' + q(t)y = 0$ where $p(t)$ and $q(t)$ are conti
functions of t, prove that for any two constants c_1 and c_2 the linear combination $c_1y_1 +$
is also a solution.

25. Find the Laplace Transform of the function $e^{at} \cos bt$.

26. Prove that the Laplace Transform is a linear operator.

27. Prove that $u(x, t) = e^{-a^2t}.\sin x$ is a solution of the equation : $a^2u_{xx} = u_t$.

28. Solve : $X' = 3X - 2Y$; $Y' = 2X - 2Y$; $X(0) = 3$ and $Y(0) = \frac{1}{2}$.

(5 × 2 =

Answer any *two* questions.

29. Find the general solution of $y'' - 3y' - 4y = 3e^{2t} + 2\sin t$.

30. Using Laplace Transforms, solve : $y'' + y = \sin 2t$; $y(0) = 2$ and $y'(0) = 1$.

31. Let $f(x) = 1 - x^2$ if $-1 \leq x \leq 1$ and $f(x+2) = f(x)$. Then

(i) Sketch the graph of the function f and state whether the function is even or odd.

(ii) Find the fourier series of f.

(iii) Deduce that : $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots = \sum_{n=1}^{\infty} \frac{1}{n^2}$.

(2 × 4 = 8)