 (Pages : 3) Name..............................

Reg. No..............................

FTH SEMESTER B.Sc. DEGREE EXAMINATION, OCTOBER 2012

(CCSS)

Mathematics

MM 5B 08—DIFFERENTIAL EQUATIONS

Three Hours Maximum : 30 Weightage

Answer *all* questions :

1. If $y(t)=e^{rt}$ is a solution of the equation $\frac{dy}{dt}+2y=0$, then $r=$ ————.

2. State whether the equation $y''+\sin(t+y)=\sin t$ is linear or non linear.

3. Give an integrating factor of the equation : $\frac{dy}{dt}+\frac{1}{2}y=2+t$.

4. State whether the equation $2x+y^2+2xyy'=0$ is exact or not.

5. Is $y=e^{3t}$ a solution of the initial value problem : $\frac{dy}{dt}=3y\,;\,y(0)=0$?

6. Write a differential equation whose general solution is $c_1e^{2t}+c_2e^{-3t}$.

7. If $f(t)=e^{\pi t}$ and $g(t)=\frac{1}{\pi}e^{\pi t}$, are the functions *f* and *g* linearly independent ?

8. If $f(t)=e^{3t}$ and $g(t)=e^{-3t}$, then the wronskian of the functions *f* and *g* is ————.

9. What is the Laplace Transforms of the function $\sin \omega t$?

10. Define the convolution product between two functions *f* and *g*.

11. The fundamental period of the function $\cos\left(\frac{\pi x}{3}\right)$ is ————.

12. State whether the function $f(x)=\left|x^3\right|$ is even or odd.

(12 × ¼ = 3 weightage)

Answer *all* questions :

13. Evaluate *b* for which the equation $\left(xy^2+bx^2y\right)dx+(x+y)x^2dy=0$ is exact.

14. Solve the initial value problem : $y'=y^2;\,y(0)=1$ and give an interval in which the solution exists.

Turn over

15. Obtain the general solution of the equation $y'' + y' + y = 0$.

16. If the wronskian $W(f,g)(t)$ of the differentiable functions f and g is non zero at som $t_0 \in I$, prove that f and g are linearly independent.

17. Find the Laplace Transform of the function $f(t) = e^{\alpha t}, t \geq 0$.

18. Find the Inverse Laplace Transform of the function $\frac{1}{s^2 - 4s + 5}$.

19. Show that the functions $\cos\left(\frac{\pi x}{L}\right)$ and $\sin\left(\frac{\pi x}{L}\right)$ are orthogonal.

20. Prove that the product of two odd functions is even.

21. Let $f(x) = 2x$ where $-1 \leq x < 1$ and $f(x+2) = f(x)$. Sketch the graph of the functi periods to which the Fourier Series of f converge.

(9 × 1 = 9 wei

III. Answer any *five* questions :

22. Prove that $\mu(x, y) = y$ is an integrating factor of the differential eq $ydx + (2x - ye^y)dy = 0$ and hence solve it.

23. Solve the initial value problem $y'' - 6y' + 9y = 0$; $y(0) = 0$ and $y'(0) = 2$.

24. If y_1 and y_2 are solutions of $y'' + p(t)y' + q(t)y = 0$ where $p(t)$ and $q(t)$ are cont functions of t, prove that for any two constants c_1 and c_2, the linear combination $c_1 y_1$ is also a solution.

25. using Laplace Transforms, solve : $y'' - y' - 2y = 0; y(0) = 0, y'(0)0$.

26. Assuming required conditions, prove that $\mathcal{L}(f'(t)) = s\mathcal{L}(f(t)) - f(0)$.

27. Find the Fourier Series of the function f given by :

$f(x) = -1$ if $-1 \leq x < 0$, $f(x) = +1$ if $0 \leq x < 1$ and $f(x+2) = f(x)$.

28. Solve : $x'_1 = 3x_1 - 2x_2$; $x'_2 = 2x_1 - 2x_2$; $x_1(0) = 3$ and $x_2(0) = \frac{1}{2}$.

(5 × 2 = 10 weig

...wer any *two* questions :

Find the general solution of $y'' - 3y' - 4y = 2\sin t$.

Let $f(x) = |x|$ if $-2 \le x \le 2$ and $f(x+4) = f(x)$. Then

(a) Sketch the graph of the function f and state whether the function is even or odd.

(b) Find the Fourier series of f.

(c) Deduce that : $\dfrac{\pi^2}{8} = 1 + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \ldots = \sum_{n=0}^{\infty} \dfrac{1}{(2n+1)^2}$.

31. Using the method of separation of variables, solve the heat equation $\alpha^2 U_{xx} = U_t$, $0 < x < L$ and $t > 0$ subject to the boundary conditions : $U(0, t) = 0 = U(L, t)$ and the initial condition $U(x, 0) = f(x)$.

(2 × 4 = 8 weightage)