D 50722 (Pages : 3) Name.....................................

Reg. No.................................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2013

(UG-CCSS)

Mathematics (Core Course)

MM 5B 05—VECTOR CALCULUS

Time : Three Hours Maximum : 30 weightage

I. Answer all the *twelve* questions :

1 Find the parametric equation of the line through the points P (–3, 2, –3) and Q (1, –1, 4).

2 Find the angle between the vectors $\bar{A} = i - 2j - 2k$, $\bar{B} = 6i + 3j + 2k$.

3 Find a vector perpendicular to both $\bar{A} = 2i + j + k$ and $\bar{B} = -4i + 3j + k$.

4 Find the equation of the plane through P_0 (–3, 0, 7) and perpendicular to $\bar{n} = 5i + 2j - \bar{k}$.

5 The equation $x = y^2 - z^2$ represents the surface of a ————

(a) Ellipsoid. (b) Cylinder.

(c) Cone. (d) Hyperbolic paraboloid.

6 Find the equation of the circular cylinder $4x^2 + 4y^2 = 9$ in cylindrical co-ordinates.

7 Find the unit tangent vector to the helix $\bar{r}(t) = \cos t\, i + \sin t\, j + tk$.

8 Find the domain and range of the function $w = \sqrt{x^2 + y^2 + z^2}$.

9 Find $\lim\limits_{\substack{(x,y)\to(1,1)\\ x\neq 1}} \left[\dfrac{xy - y - 2x + 2}{x - 1}\right]$.

10 If $w = x^2 + y^2 - z + \sin t$ and $x + y = t$, find $\left(\dfrac{\partial w}{\partial y}\right)$ and $\left(\dfrac{\partial w}{\partial x}\right)$.

Turn over

11 Find the gradient of $g(x, y, z) = e^z - \ln(x^2 + y^2)$

12 State the Fubini's theorem (first form).

(12 × ¼ = 3 weightage)

II. Answer *all* the nine questions :

13 Find the point where line $x = \frac{8}{3} + 2t, \quad y = -2t, \; z = 1 + t$ intersects the plane $3x + 2y + 6z = 6$.

14 Find the spherical and cylindrical equation of the hemisphere $x^2 + y^2 + (z - 1)^2 = 1, \; z \le 1$.

15 Show that $\bar{u}(t) = \sin t\, i + \cos t\, j + \sqrt{3}\, k$ has a constant length and is orthogonal to its derivative.

16 Show that the function $f(x, y, z) = 2z^3 - 3(x^2 + y^2)\, z$ satisfies the Laplace's equation.

17 Find the derivative of $f(x, y) = xe^y + \cos(xy)$ at the point (2, 0) in the direction of $\bar{A} = 3i - 4j$.

18 Find the saddle point if any of the function $f(x, y) = x^2 + xy + 3x + 2y + 5$.

19 Calculate $\iint_R \frac{\sin x}{x}\, dA$ where R is the triangle in the xy plane bounded by the x-axis, the line $y = x$ and the line $x = 1$.

20 Find the work done by $\bar{F} = xyi + yj - yzk$ over the curve $\bar{r}(t) = ti + t^2 j + tk, \; 0 \le t \le 1$.

21 Evaluate $\int_0^1 \int_0^{1-z} \int_0^2 dx\, dy\, dz$.

(9 × 1 = 9 weightage)

III. Answer any *five* questions from seven :

22 Find the unit tangent vector, normal vector and binormal for the curve $\bar{r}(t) = (\cos t + t \sin t)\, i + (\sin t - t \cos t)\, j + 3k$.

23 Find the linearization of $f(x, y) = x^2 - xy + \frac{y^2}{2} + 3$ at the point (3, 2)

24 Find the derivative of $f(x, y, z) = ln(2x + 3y + 6z)$ at $p(-1, -1, 1)$ in the direction of $\bar{A} = 2i + 3j + 6k$.

25 Find the average value of $F(x, y, z) = x^2 + y^2 + z^2$ over the cube in the first octant bounded by the co-ordinate planes and the planes $x = 1, y = 1$ and $z = 1$.

26 Show that $\bar{F} = (y + z)i + (x + z)j + (x + y)k$ forms a conservative force field and find its potential function.

27 Apply Green's theorem to evaluate $\oint_C (y^2\, dx + x^2\, dy)$ where C is the triangle bounded by $x = 0, x + y = 1, y = 0$.

28 Integrate $g(x, y, z) = xyz$ over the surface of the cube cut-off by the first octant by $x = 1, y = 1, z = 1$.

(5 × 2 = 10 weightage)

IV. Answer any *two* questions :

29 Find the local extreme values of the function $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$.

30 Use Taylor's theorem for $f(x, y)$ to find a quadratic and cubic approximation of $f(x, y) = x\, e^y$ at origin.

31 Use Stoke's theorem to evaluate $\int_C \bar{F} \cdot d\bar{r}$ if $\bar{F} = xzi + xyj + 3x\, zk$ where C is the boundary of the portion of the plane $2x + y + z = 2$ is the first octant traversed in counterclockwise sense.

(2 × 4 = 8 weightage)