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Name.......................................

Reg. No.................................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2013

(U.G.–CCSS)

Mathematics—Core Course

MM 5B 06—ABSTRACT ALGEBRA

Time : Three Hours

Maximum : 30 Weightage

Answer all *twelve* questions.

1. Define an abelian group.
2. Order of a finite group G is ————.
3. In the addition group of integers the order of every element zero is ————.
4. Give an example of a cyclic group.
5. State True *or* False :

   If $a \in H = H$ then Ha = aH = H, where H is a subgroup of a group.
6. S_n has ———— elements.
7. State True *or* False :

   Every permutation is a one-to-one function.
8. Define a transposition.
9. State True *or* False :

   The set φ of rational numbers is not a ring w.r.t. ordinary addition and multiplication.
10. Write the smallest subspace of any vector space.
11. Is it true that if the set $S = \{u_1, u_2, \ldots, u_k\}$ of a vector space V is L.D. then every superset of S is also L.D. ?
12. Define the basis of a vector space.

(12 × ¼ = 3 weightage)

Short answer questions.
Answer *all* questions.

13. What is the dimension of a vector space.
14. Define the span of a set.
15. Give an example of a linearly independent set.

Turn over

16. Define a homomorphism.
17. Give an example of a field.
18. What is the order of $\mu = (1,4)(3,5,7,8)$.
19. Consider $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}, \beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 3 & 6 & 5 \end{pmatrix}$, compute $\alpha\beta^2$.
20. Show that $\{1, -1, i, -i\}$ is a cyclic group.
21. Examine whether $G = \{-3, -2, -1, 0, 1, 2, 3\}$ for the operation +, is a group.

(9 × 1 = 9 weightage)

Answer any *five* questions.

22. Prove that the set $G = \{a + b\sqrt{2} : a, b, \in R\}$ forms a group under multiplication.
23. Prove that if a, b are any two elements of a group G then $(a \cdot b)^2 = a^2b^2$ iff G is abelian.
24. Prove that every group of prime order is cyclic.
25. Let V = R_3 be the vector space. Let $U = \{u = (x_1, x_2, x_3) \in V \mid x_1 + x_2 + x_3 = 0\}$. Show that U is a subspace of V.
26. Check whether the set :

    $S = \{(1,1,0), (1,0,1), (0,1,1)\}$.

    linearly independent in V_3.
27. If p is prime then prove that Z_p is a field.
28. Prove that if a finite group of order n contains an element of order n then the group must be cyclic.

(5 × 2 = 10 weightage)

Answer any *two* questions.

29. Show that a non-empty subset H of a group G is a subgroup of G iff $ab^{-1} \in H$ for all $a, b \in H$.
30. Prove that every finite integral domain is a field.
31. Suppose $S = \{V_1, V_2, \ldots, V_k\}$ is an ordered set of a vector space V. If $V_1 \neq 0$ then prove that the set S is L.D. iff one of the vectors of $\{V_1, V_2, \ldots, V_k\}$ belongs to the span of remaining other vectors of the set S.

(2 × 4 = 8 weightage)