D 50599 (Pages : 3) Name.................................

Reg. No.................................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(CUCBCSS–UG)

MAT 5B 05—VECTOR CALCULUS

Time : Three Hours Maximum : 120 Marks

Part A

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. Find the domain and range of $f(x,y) = \sin xy$.
2. Evaluate $\lim_{(x,y)\to(0,0)} \frac{x}{y}$.
3. Find the gradient of $\phi(x,y,z) = xyz$.
4. Compute the divergence of $\vec{f} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$.
5. In what direction the directional derivative of a function becomes maximum ?
6. What do you mean by an irrotational vector ?
7. What is the linearization of the function $f(x,y,z)$ at the point (1, 2, 3) ?
8. Find the total differential of u if $u = \ln\left(x^2 + y^2 + z^2\right)$.
9. Fill in the blanks : If $\vec{f}$ and $\vec{g}$ are irrotational vector point functions, then $\nabla\cdot\left(\vec{f}\times\vec{g}\right) = \ldots$
10. State the Normal form of Green's theorem in the plane.
11. Fill in the blanks : If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, then $r^n\vec{r}$ is solenoidal if $n = \ldots.$
12. State Gauss's Divergence theorem mentioning all the assumptions involved in it explicitly.

(12 × 1 = 12 marks)

Part B

Answer any **ten** *questions.*
Each question carries 4 *marks.*

13. Find the vector normal to the surface $\phi(x,y,z) = x^3 + y^3 + z^3$ at (1, –1, 1).
14. Evaluate $\lim_{(x,y)\to(0,0)} \frac{y-x}{y+x}$.

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15. If $z^3 - xy + yz + y^3 - 2 = 0$ then, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at (1, 1, 1).

16. Find the total derivative of $u = x^3 + y^3$ with respect to t if $x = \cos t, y = \sin t$.

17. Compute the average value of the function $f(x, y, z) = xyz$ over the boundary of the cube $0 \le x \le 2, 0 \le y \le 2, 0 \le z \le 2$.

18. Linearize the function $f(x,y,z) = x^2 - xy + 3\sin z$ at (2,1,0).

19. Find the directional derivative of $f(x,y,z) = xy + yz + zx$ at (1, 2, 3) in the direction of $3\bar{i} + 4\bar{j} + 5\bar{k}$.

20. Find the circulation of $\bar{f} = (x,y)\bar{i} + x\bar{j}$ around the unit circle centered at the origin.

21. Find the value of λ which makes the following vector $\bar{f} = \left(\lambda xy - z^3\right)\bar{i} + \left((\lambda - 2)x^2\right)\bar{j} + \left((1-\lambda)xz^2\right)\bar{k}$ is irrotational.

22. Verify whether the differential $e^x \cos y dx + \left(xz - e^x \sin y\right)dy + (xy + z)dz$ is exact or not.

23. If $\bar{f}$ and $\bar{g}$ are irrotational, then show that $\bar{f} \times \bar{g}$ is solenoidal.

24. Evaluate $\int_0^2 \int_{x^2}^{2x} (4x + 2)\, dy dx$ by changing the order of integration.

25. Prove that if a function $f(x, y)$ is differentiable at the origin, then it is continuous at the origin.

26. Evaluate the integral $\int_c xy dy - y^2 dx$ where c is the square cut from the first quadrant by the lines $x = 1$ and $y = 1$.

(10 × 4 = 40 marks)

Part C

Answer any **six** *questions.*
Each question carries 7 marks.

27. Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy dx$.

28. Find $\bar{\nabla} f(r)$, if $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$.

29. Show that the work done by the force field $\bar{f} = yz\bar{i} + zx\bar{j} + xy\bar{k}$ is independent of the path joining the points (–1, 3, 9) and (1,6, – 4).

30. Test the continuity of $f(x, y)$ defined by $f(x,y) = \frac{x^2+y^2}{x+y}, (x,y) \neq (0,0)$ and $f(x, y) = 0$, $(x, y) = (0, 0)$.

31. Find the equation to the tangent plane and normal line to the surface $f = (x, y, z) = x^2 + y^2 + z^2 - 9 = 0$ at the point $(1, 2, 4)$.

32. Evaluate the area encosed by the region $x^2 + y^2 = 4, y = 1, y = \sqrt{3x}$ in the (x, y) plane, using double integrals.

33. Find the Local extreme values of $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$.

34. Evaluate the volume of the ellipsoide $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

35. Show that $\vec{f} = (x + 2y + 4z)\vec{i} + (2x - 3y - z)\vec{j} + (4x - y - 2z)\vec{k}$ is conservative and find its scalar potential.

(6 × 7 = 42 marks)

Part D

Answer any **two** *questions.*
Each question carries 13 *marks.*

36. (a) Verify Gauss's divergence theorem for $\vec{f} = x\vec{i} + y\vec{j} + z\vec{k}$ over the sphere of radius a centered at the origin.

(b) State the Fundamental theorem of line integration.

37. (a) Evaluate the line integral $\int_C \vec{f} \cdot \overrightarrow{dr}$ where C is the boundary of the triangle with vertices (0,0,0), (1,0,0) (1,1,0).

(b) Find (Curl Curl) $\vec{f}$, if $\vec{f} = = x^2 y\vec{i} - 2xz\vec{j} + 2yz\vec{k}$.

38. (a) If S is a closed surface enclosing a volume V then prove that $\iint_S \text{curl}\,\vec{f} \cdot \hat{n}\, d\,S = 0$.

(b) Verify Green's theorem for $\int_C (xy + y^2)dx + x^2 dy$, where C is the curve joining $y = x^2$ and y =

(2 × 13 = 26 mark