172 (Pages : 2) Name..............................

Reg. No..............................

[F]IFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2011

(CCSS)

Mathematics—Core Course

MM 5B 05—VECTOR CALCULUS

[Time] : Three Hours Maximum Weightage : 30

Answer all the **twelve** *questions.*

A vector that is perpendicular to both of the vectors $\bar{A}$ and $\bar{B}$ is ——————.

Find parametric equations of the line through the points P (–2, 0, 3) and Q (3, 5, –2).

Write a vector normal to the plane $ax + by + cz = d$.

The Cartesian equation of the surface $z = r^2$ is —————.

A particle moves along the curve $x = 3t^2, y = t^2, z = t^3$. Find the velocity at $t = 1$.

The curvature of a straight line is —————.

Domain of the function $f(x, y, z) = \dfrac{1}{xyz}$ is —————.

Find dy/dx if $x^2 + \sin y - 2y = 0$.

Find the gradient of the function $f(x, y) = y - x$ at (2, 1).

Write the Taylor's formula for $f(x, y)$ at the origin.

State Fubini's theorem (first form) for calculating double integrals.

Find the divergence of $\vec{F} = 2xz\,\vec{i} - xy\,\vec{j} = z\,\vec{k}$.

(12 × ¼ = 3)

Answer all the **nine** *short answer questions.*

Find the angle between the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$.

Find the spherical coordinate equation for the sphere $x^2 + y^2 + (z - 1)^2 = 1$.

Show that the function $f(x, y, z) = x^2 + y^2 - 2z^2$ satisfies the Laplace's equation $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} + \dfrac{\partial^2 f}{\partial z^2} = 0$.

Find the derivative of $xe^y + \cos(xy)$ at (2, 0) in the direction of $\vec{A} = 3\,\vec{i} - 4\,\vec{j}$.

Find the local extreme values of the function $f(x, y) = xy$.

Find the area enclosed by the lemniscate $r^2 = 4\cos 2\theta$.

Evaluate $\displaystyle\int_1^e\int_1^e\int_1^e \frac{1}{xyz}\,dz\,dy\,dx$.

Turn over

20. Find the circulation of the field $\vec{F} = (x - y)\,\vec{i} + x\vec{j}$ around the $\overrightarrow{r(t)} = \cos t\ \vec{i} + \sin \vec{j}, 0 \le t \le 2\pi$.

21. Find the divergence of $\vec{F} = (x^2 - y)\,\vec{i}\ (xy - y^2)\,\vec{j}$.

(9 ×

Answer any **five** *short essay questions.*

22. Find the unit tangent vector, Principal unit normal vector, Binormal vector curvature and at t for the curve $\overrightarrow{r(t)} = 3\sin t\ \vec{i} + 3\cos t\ \vec{j} + 4t\ \vec{k}$.

23. Find the linearization of $f(x, y) = x^2 - xy + \frac{1}{2}y^2 + 3$ at the point of (3, 2).

24. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, –1, 2

25. Find the average value of $\vec{F}(x, y, z) = xyz$ over the cube bounded by the coordinate pla the planes $x = 2$, $y = 2$ and $z = 2$ in the first octant.

26. Show that $\vec{F}\ (e^x \cos y + yz)\,\vec{i} + (xz - e^x \sin y)\,\vec{j} + (xy + z)\,\vec{k}$ is conservative and find a po function for it.

27. Using Green's theorem evaluate the integral $\oint_C xydy - y^2dx$ where C is the square cut fr first quadrant by the lines $x = 1$, and $y = 1$.

28. Find a parametrization of the cone $z = \sqrt{x^2 + y^2}, 0 \le z \le 1$.

(5 × 2

Answer any **two** *essay questions.*

29. Find the points closest to the origin on the hyperbolic cylinder $x^2 - z^2 - 1 = 0$.

30. Evaluate $\int_0^1 \int_0^{1-x} \sqrt{x + y}\,(y - 2x)^2\, dydx$ by applying the transformations $u = x + y$ and $v =$

31. State Divergence theorem. Verify Divergence theorem for the field $\vec{F} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$ ov sphere $x^2 + y^2 + z^2 = a^2$.

(2 × 4