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Name.......................................

Reg. No...................................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, OCTOBER 2012

(CCSS)

Mathematics (Core)

MM 5B 05—VECTOR CALCULUS

Three Hours Maximum : 30 Weightage

Answer **all** *the twelve questions.*

Find the unit vector obtained by rotating $\bar{i}$ clockwise through an angle $\pi/4$.

The value of $(A \times B) \cdot A$ is ———.

The vector $\bar{r}(t) = 3\cos t\,\bar{i} + 3\sin t\,\bar{j} + t^2\bar{k}$ gives the position of a moving body at time t. Find the body's speed at $t = 2$.

Find all level curves of the function $f(x, y) = 100 - x^2 - y^2$.

At what points (x, y) in the plane the function $\sin(x + y)$ is continuous.

Find the equation for the cylinder $x^2 + (y - 3)^2 = 9$ in cylindrical coordinates.

If $\bar{u}$ is a differentiable function of t of constant length, then $u \cdot \frac{du}{dt} =$ ———.

Find $\dfrac{\partial f}{\partial y}$ if $f(x, y) = y \sin xy$.

Evaluate $\int_0^1 \int_0^1 (x^2 + y^2)\,dx\,dy$.

The double integral form of the area of a closed and bounded region R in the polar coordinate plane is ———.

Find the gradient field of $f(x, y, z) = xyz$.

State Stoke's theorem.

$(12 \times \frac{1}{4} = 3$ Weightage)

Answer **all** *the nine short answer questions.*

Find a vector parallel to the plane $2x - y - z = 4$ and orthogonal to $i + j + k$.

Find the length of one turn of the helix $\overline{r(t)} = \cos t\,\bar{i} + \sin t\,\bar{j} + t\,\bar{k}$.

Turn over

15. Show that

$$f(x, y) = \begin{cases} \dfrac{2xy}{x^2 + y^2} & \text{when } (x, y) \neq (0, 0) \\ 0, & \text{when } (x, y) = (0, 0) \end{cases}$$

is continuous at every point except the origin.

16. Find the plane tangent to the surface $z = x\cos y - ye^x$ at (0, 0, 0).

17. Using Taylor's formula find a quadratic approximation for $f(x, y) = \sin x \sin y$ near th

18. Find the area enclosed by the lemniscate $r^2 = 4\cos 2\theta$.

19. Evaluate $\int_1^e \int_1^e \int_1^e \frac{1}{xyz}\, dz\,dy\,dx$.

20. Evaluate $\int_C (x - 3y^2 + z)\,ds$ where C is the line segment joining the origin and the point

21. Find the divergence of $\bar{F} = (x^2 - y)\,\bar{i} + (xy - y^2)\,\bar{j}$.

(9 × 1 = 9

Answer any **five** *short essay questions out of seven.*

22. Find the unit tangent vector, Principal unit normal vector, Binormal vector curvature

at t for the curve $\overline{r(t)} = (e^t \cos)\,\bar{i} + (e^t \sin t)\,\bar{j} + 2\,\bar{k}$.

23. Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x^2 + y^2$, $x = r - s$, $y = r + s$.

24. Find the derivative of $f(x, y) = xe^y + \cos(xy)$ at the point (2, 0) in the direction of A =

25. Find the local extreme values of $f(x, y) = xy$.

26. Calculate $\iint_R \frac{\sin x}{x}\,dA$ where R is the triangle in the xy-plane bounded by the x-axis,

$y = x$, and the line $x = 1$.

7. Show that $\bar{F} = (e^x \cos y + yz)\,\bar{i} + (xz - e^x \sin y)\,\bar{j} + (xy + z)\,\bar{k}$ is conservative and find a potential function for it.

8. Using Green's theorem evaluate the integral $\oint_C xydy - y^2dx$ where C is the square cut from the first quadrant by the lines $x = 1$, and $y = 1$.

(5 × 2 = 10 Weightage)

Answer any **two** *Essay questions out of three.*

9. Find the absolute maximum and minimum values of $f(x, y) = 2 + 2x + 2y - x^2 - y^2$.

0. Find the volume of the solid enclosed by the sphere $x^2 + y^2 + z^2 = a^2$.

1. State Divergence theorem. Verify Divergence theorem for the field $\bar{F} = x\,\bar{i} + y\,\bar{j} + z\,\bar{k}$ over the sphere $x^2 + y^2 + z^2 = a^2$.

(2 × 4 = 8 Weightage)