D 30557 **(Pages : 2)** **Name..................................**

Reg. No..................................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, OCTOBER 2012

(CCSS)

Mathematics

MM 5B 06—ABSTRACT ALGEBRA

Time : Three Hours Maximum : 30 Weightage

Questions from **1 to 12** are **compulsory.** Each has weightage ¼.

1. How many generators are there for the group Z_4 ?
2. Write the number of orbits in the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 8 & 6 & 7 & 4 & 1 & 5 & 2 \end{pmatrix}$
3. Give a proper subgroup for the group R^+ under multiplication.
4. A cyclic group with only one generator can have at most ——— elements.
5. The order of the alternating group A_4 is ———.
6. The number of cosets of the subgroup 4Z in Z is ———.
7. State True *or* False : "Every group of order 31 is cyclic".
8. Let $\phi : Z \to R$ under addition be given by $\phi(n) = n$. Is ϕ a homomorphism ?
9. A non-commutative division ring is called ———.
10. Is 2 a unit element of the ring Q ?
11. Write the number of zero divisors of the field Z_5.
12. What is the dimension of the vector space R^2 over R.

(12 × ¼ = 3 weightage)

Short Answer Type Questions : (Answer *all* questions).

13. Prove that the set $M_2(R)$ of all 2 × 2 matrices with operation of matrix multiplication is not a group.
14. Let G be a group and let $a \in G$. Then prove that $H = \{a^n / n \in Z\}$ is a subgroup of G.
15. Write any five elements of the cyclic group 25Z.
16. Show that the permutation $(1, 4, 5, 6)(2, 1, 5)$ is an odd permutation.
17. Find the number of generators of the cyclic group of order 8.

Turn over

18. If G is a group and let $\phi : G \to G$ be given by $\varphi(g) = g^{-1}$. Is ϕ a homomorphism ?
19. Define the terms unit element, division ring and skew field.
20. Check whether the set $\{(1,1,0),(1,0,1),(0,1,1)\}$ is linearly independent in R^3 or
21. Is $Q(R)$ a vector space ? Verify.

(9 × 1 =

Short Essay Questions : (Answer any *five* questions out of seven)

22. Show that every group G with identity e such that $a * a = e \ \forall a \in G$ is abelian.
23. Prove that subgroup of a cyclic group is cyclic.
24. Define a finitely generated group. List all elements of the subgroup of Z_{12} generated b $\{2, 3\}$.
25. Define the term group homomorphism. If $\phi : G \to G'$ be a group homomorphism of prove that G' is abelian if G is abelian.
26. Show that $\{1, x, x^2\}$ forma basis for $P_2(x)$, the set of all real polynomials in x of degre
27. Prove that every field is an Integral Domain.
28. Prove that intersection of two subspaces of a vector space is again a subspace.

(5 × 2 = 10

Essay Questions : (Answer any *two* questions out of three)

29. (a) Define an abelian group.
    (b) Describe Klein 4-group V. Is it an abelian group ? Verify.
    (c) Write all subgroups of V. Specify which are abelian.
30. (a) Define a cyclic group. Give an example.
    (b) Prove that every cyclic group is abelian.
    (c) Given an example of an infinite group which is not cyclic.
    (d) Find all subgroup if Z_{12}.
31. (a) State and prove Langrage theorem.
    (b) Prove that every group of prime order is cyclic.
    (c) Prove that every group of order 17 is abelian.

(2 × 4 = 8