

D 50601

(Pages : 3)

Name.....

Reg. No.....

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(CUCBCSS—UG)

MAT 5B 07—BASIC MATHEMATICAL ANALYSIS

Time : Three Hours

Maximum : 120 Marks

Part A

Answer all the twelve questions.
Each question carries 1 mark.

1. Fill in the blanks : Supremum of the set $S = \{1/m - 1/n ; m, n \in \mathbb{N}\}$ is _____.
2. Determine the set $A = \{x \in \mathbb{R} : |2x + 3| < 7\}$.
3. The Set of all real numbers which satisfy the inequality $0 \leq a < \epsilon, \forall \epsilon > 0$, then $a =$ _____.
4. Fill in the blanks : The Supremum property of \mathbb{R} states that _____.
5. State the Trichotomy Property of \mathbb{R} .
6. Give the condition for a subset of \mathbb{R} to be an Interval of \mathbb{R} .
7. State the general Arithmetic-Geometric mean Inequality of real numbers .
8. Fill in the blanks : The Characterization Theorem of Open sets states that _____.
9. State the Archimedian Property of positive Integers.
10. If $c > 0$, then $\lim (c^{1/n}) =$ _____.
11. State the Bolzano-Weierstrass Theorem on Sequence .
12. Fill in the blanks : The Exponential form of $-1 - i =$ _____.

(12 × 1 = 12 marks)

Part B

Answer any ten questions.
Each question carries 4 marks.

13. State and prove Bernoulli's Inequality of Real numbers.
14. Show that there doesnot exist a rational number r such that $r^2 = 2$.
15. If $a, b \in \mathbb{R}$, then prove that $||a| - |b|| \leq |a - b|$.

Turn over

16. Prove that a real sequence can have at most one limit.
17. If $x \in \mathbb{R}$ then prove that there exists $n_x \in \mathbb{N}$ such that $x < n_x$.
18. State and prove the "Betweenness Property" of Irrational numbers.
19. If $X = (x_n)$ is a convergent sequence of real numbers satisfying $\lim (x_n) = x$, then show that $\lim (|x_n|) = |x|$.
20. Prove that the set of irrational numbers is uncountable.
21. Let A and B be bounded non-empty subsets of real numbers such that $a \leq b, \forall a \in A, b \in B$. Prove that $\text{Sup } A \leq \text{Inf } B$.
22. Discuss the convergence of $X = (x_n)$ defined by $x_n = n$, if n odd and $x_n = 1/n$, if n even.
23. Show that every bounded sequence of real numbers has a converging sub-sequence.
24. Test the convergence of the sequence $\left(\frac{\sin n}{n}\right)$.
25. Define Cauchy sequence test whether $(1/n)$ is a Cauchy sequence or not.
26. Find all values of $(-8i)^{1/3}$.

(10 × 4 = 40 marks)

Part C

*Answer any six questions.
Each question carries 7 marks.*

7. State and prove the Nested Interval Property.
8. Define denumerable set. Show that the set \mathbb{Q} of rational numbers is denumerable.
9. If the set A_m is countable for each $m \in \mathbb{N}$, then prove that $A = \bigcup_{m=1}^{\infty} A_m$ is countable.
10. $X = x_n$ and $Y = y_n$ be sequences of real numbers converges to x and y respectively, then prove that $X \cdot Y$ converges to xy .

31. (a) Give an example of a convergent sequence (x_n) of positive real numbers with $\lim \left(\frac{x_{n+1}}{x_n} \right) = 1$.
- (b) Give an example of a divergent sequence (x_n) of positive real numbers with $\lim \left(\frac{x_{n+1}}{x_n} \right) = 1$.
- (c) Give your comments about the property of the sequence (x_n) of positive real numbers with $\lim \left(\frac{x_{n+1}}{x_n} \right) = 1$.
32. If $X = (x_n)$ is a real sequence and $X_m = (x_{n+m} : n \in \mathbb{N})$ is the m -tail of X ; $m \in \mathbb{N}$, then show that X_m converges to x if and only if X converges to x .
33. Find a sequence (x_n) of real numbers such that $\lim |x_{n+1} - x_n| = 0$, but not a Cauchy sequence.
34. (a) Find the Arg Z , if $Z = \frac{i}{-2-2i}$.
- (b) Express the complex number $(\sqrt{3+i})^7$ in Rectangular form.
35. Discuss the convergence of the following sequences whose n 'th terms are defined by
- (a) $x_n = \left(1 + \frac{1}{n^2}\right)^{2n^2}$ and (b) $y_n = \frac{\log n}{n}$.

(6 × 7 = 42 marks)

Part D

Answer any **two** questions.
Each question carries 13 marks.

36. State and prove the Cauchy convergence criterion for sequence.
37. (a) Show that the unit interval $[0,1]$ is uncountable.
- (b) State and prove the Ratio Test for the convergence of real sequence.
38. (a) Define closed sets in \mathbb{R} . Show that the Intersection of an arbitrary collection of closed sets in \mathbb{R} is closed.
- (b) Show by an example that the union of infinitely many closed sets in \mathbb{R} need not be closed.

(2 × 13 = 26 marks)