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Name.......................................

Reg. No.......................................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(CUCBCSS—UG)

Mathematics

MAT 5B 07—BASIC MATHEMATICAL ANALYSIS

Time : Three Hours

Maximum : 120 Marks

Part A

Answer all the **twelve** *questions.*
Each question carries 1 mark.

1. Fill in the blanks : Infimum of the $S = \{1/m - 1/n; m, n \in N\}$ is ————.
2. The Set of all real numbers which satify the inequality $\left|x^2 - 1\right| \leq 3$ is ————.
3. Fill in the blanks : The $\in$ neighborhood of $a \in R$ is ————.
4. State the Infimum Property of R.
5. State Bernoulli's Inequality.
6. Fill in the blanks : If X is a converging sequence of non-negative real numbers, then $\lim X =$ ————.
7. Fill in the blanks : The intersection of infinitely many open sets in R is ————.
8. State the Density Theorem.
9. State the Monotone Sub-sequence Theorem.
10. Give an example of a bounded real sequence which is not a Cauchy sequence.
11. Fill in the blanks : The Polar form of $1 + i\sqrt{3} =$ ————.
12. Find the ArgZ, if $Z = \frac{-2}{1 + i\sqrt{3}}$.

(12 × 1 = 12 marks)

Part B

Answer any **ten** *questions.*
Each question carries 4 marks.

13. Define Supremum and Infimum of set. Find them for the Set $S = \left\{1 - \frac{(-1)^n}{n}; n \in N\right\}$.
14. Prove that the set N of positive integers is not bounded above.

Turn over

15. If a is a real number such that $0 \le a \le \varepsilon$ for every $\varepsilon > 0$, then prove that $a = 0$.

16. Show that there does not exist a rational number r such that $r^2 = 5$.

17. State and prove Squeeze theorem on sequence.

18. If $S = \{1/n ; n \in N\}$, then prove that Infimum of $S = 0$.

19. If X and Y are convergent sequences of real numbers satisfying $(x_n) \le (y_n), \forall n \in N$, then prove that $\lim(x_n) \le \lim(y_n), \forall n \in N$.

20. Prove that every bounded sequence of real numbers has a converging sub-sequence.

21. Define Cauchy sequence. Show that $(1/n)$ is a Cauchy sequence.

22. Prove that every converging sequence is a Cauchy sequence.

23. Prove or disprove that "the union of infinitely many closed sets in R is closed ".

24. Let S and T be bounded non-empty subsets of real numbers such that $S \subset T$. Prove that $\text{Inf } T \le \text{Inf } S \le \text{Sup } S \le \text{Sup } S$.

25. Test the convergence of the sequence $\left(\frac{\log n}{n}\right)$.

26. Find all values of $(-27i)^{\frac{1}{3}}$.

(10 × 4 = 40 marks)

Part C

Answer any **six** *questions.*
Each question carries 7 marks.

27. State and prove Characterization theorem of Intervals.

28. Prove that [0, 1] is uncountable.

29. State and prove the "Betweeness Property" of Irrational numbers.

30. Determine the set $A = \left\{x \in R : \frac{2x+1}{x+2} < 1\right\}$.

31. Let $X = (x_n)$ be a non-negative sequence of real numbers with $\lim(x_n) = x$. Prove that $\lim\left(\sqrt{x_n}\right) = x$.

32. (a) Give an example of a convergent sequence (x_n) of positive real numbers with $\lim(x_n)^{\frac{1}{n}} = 1$.

(b) Give an example of a divergent sequence (x_n) of positive real numbers with $\lim(x_n)^{\frac{1}{n}} = 1$.

(c) Justify the property of the sequence (x_n) of positive real numbers with $\lim(x_n)^{\frac{1}{n}} = 1$.

33. Test whether the (x_n) defined by $x_n = 1 + 1/2 + 1/3 + \ldots + 1/n$ is Cauchy sequence or not.

34. Prove that every contractive sequence is a Cauchy sequence.

35. Give an algebraic proof for the triangle inequality of complex numbers,

$$|(z_1 + z_2)| \le |(z_1)| + |(z_2)|, \forall z_1, z_2 \in \mathbb{C}.$$

(6 × 7 = 42 marks)

Part D

Answer any **two** *questions.*
Each question carries 13 *marks.*

36. If $I_n = [a_n, b_n], n \in \mathbb{N}$ is a nested sequence of closed and bounded intervals, then prove that there exist a common point in every I_n.

37. State and prove the monotone convergence theorem of sequence.

38. (a) State and prove the Ratio Test for the convergence of real sequence.

(b) Define "cluster point" of a set. Prove that a subset of R is closed in R if and only if it contains all of it's cluster points.

(2 × 13 = 26 marks)