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Name..................................

Reg. No..................................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2011

(CCSS)

Mathematics—Core Course

MM 5B 07—BASIC MATHEMATICAL ANALYSIS

Three Hours

Maximum Weight : 30

Answer **all** *questions, weight* $\frac{1}{4}$ *each.*

The symmetric difference $A \Delta B =$ ———.

The property of $\mathbb{N}$ '*Every nonempty subset of* $\mathbb{N}$ *has a least element*' is known as ———.

——— is an example of a bounded sequence which is not convergent.

$\lim_{n \to \infty} n^{1/n} =$ ———.

The $\frac{1}{2}$-neighbourhood of 2 in $\mathbb{R}$ is ———.

Let $I_n = [0, 1/n)$, then $\bigcup_{n=1}^{\infty} I_n =$ ———.

——— is an example of an open subset of $\mathbb{R}$, which is not an open interval.

Let $A = \{1/n : n \in \mathbb{N}\}$. Then the interior $\mathring{A} =$ ———.

Let $A \subset \mathbb{R}$. The smallest closed set containing A is ———.

The complement of an open set is ———.

Geometrically the set $\{z \in \mathbb{C} : |z| = 1\}$ is ———.

$\arg(z_1 \cdot z_2) =$ ———.

$(12 \times \frac{1}{4} = 3)$

Short Answer Type Questions (*Answer* **all** *questions, weight* 1 *each.*

If the functions $f : A \to B$ and $g : B \to C$ are bijective, show that $g \circ f$ is bijective.

State the principle of strong induction.

Define positive real numbers.

Evaluate $\lim_{n \to \infty} \frac{2n+1}{n+5}$.

Turn over

17. Prove by an example that sum of two divergent sequences need not be divergent.

18. If X = (x_n) is a convergent sequence such that $a \leq x_n \leq b$ for all $n \in \mathbb{N}$, show $a \leq \lim_{n\to\infty} x_n \leq b$.

19. Show that subsequence of a convergent sequence is convergent.

20. Prove or disprove : arbitrary intersection of open sets is open.

21. Find the principal argument of $(\sqrt{3} - i)^6$.

(9 × 1 =

Short Essay or Paragraph Questions : Answer any **five** *questions, weight 2 each.*

22. Using Mathematical induction, prove that $5^{2n} - 1$ is divisible by 8 for all $n \in \mathbb{N}$.

23. Show that Q is denumerable.

24. Prove the Bernoulli's inequality : If $x > -1$, then $(1 + x)^n \geq 1 + nx, \forall n \in \mathbb{N}$.

25. If the sequences X = (x_n) and Y = (y_n) converges to x and y respectively, then show that sequence XY = $(x_n y_n)$ converges to xy.

26. Let (x_n) be a bounded sequence and let $s = \sup\{x_n : n \in \mathbb{N}\}$. Show that if $s \neq x_n$ for any n, th there is a subsequence of (x_n) that converges to s.

27. Show that a subset of $\mathbb{R}$ is closed if and only if it contains all of its cluster points.

28. If z_1, z_2, z_3 be three complex numbers such that $z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1$. Show th $|z_1 - z_2| = |z_2 - z_3| = |z_3 - z_1|$.

(5 × 2 =

Essay Questions : Answer any **two** *questions, weight 4 each.*

29. (a) Prove Cantor's Theorem.

    (b) Prove nested interval property of $\mathbb{R}$.

30. State and prove Bolzano-Weierstrass Theorem. (Second proof).

31. Show that a subset F of $\mathbb{R}$ is closed if and only if every convergent sequence in F has limit in

(2 × 4 =