D 30558 (Pages : 2) Name..............................

Reg. No..............................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, OCTOBER 2012

(CCSS)

Mathematics

MM 5B 07—BASIC MATHEMATICAL ANALYSIS

Time : Three Hours Maximum : 30 Weightage

Answer **all** *questions from* 1 *to* 12, *weightage* $\frac{1}{4}$ *each.*

1. If A and B are two infinite sets then A – B is finite. True *or* False ?
2. The Inequality $(1+x)^n \geq 1+nx$ is known as ———.
3. For $a \in \mathbb{R}$ and $\epsilon > 0$ the ϵ– neighbourhood $V_\epsilon(a) =$ ———
4. A positive real number is rational if and only if its decimal representation is ———.
5. $\lim_{n \to \infty} \left(\frac{2n+9}{n+3}\right) =$ ———.
6. $\lim_{n \to \infty} (1+1/n)^n =$ ———.
7. Give an example of a divergent sequence which is ultimately convergent.
8. Give a divergent sequence that is not properly divergent.
9. Give an example of a set A such that every element of A is a boundary point of A.
10. Give an example of a set having no cluster point.
11. State Euler's formula.
12. $\arg(z_1 / z_2) =$ ———.

($12 \times \frac{1}{4} = 3$ Weightage)

Short Answer Type Questions (Answer **all** *questions, weightage* 1 *each.*

13. State Archimedean property of real numbers.
14. State density theorem for national numbers in $\mathbb{R}$.
15. Define ultimate property of sequence.
16. State Bolzano-Weierstrass Theorem.
17. Define Cauchy sequence.
18. Define Contractive sequence.
19. Prove or disprove : Arbitrary union of closed sets is closed.

Turn over

20. If $a \in \mathrm{R}$ and $a \neq 0$, then show that $a^2 > 0$.

21. Show that $\arg \bar{z} = -\arg z$.

(9 × 1 = 9 Weight

Short Essay or Paragraph Questions, Answer any **five** *questions seven, weightage 2 each.*

22. Using mathematical induction prove that if the set S has n elements, the powerset $\mathcal{P}(S)$ ha elements.

23. Show that a sequence $X = (x_n)$ is convergent if and only if the m-tail X_m is convergent for $m \in \mathbb{N}$.

24. Show that $\lim_{n \to \infty} \left(\frac{\sin n}{n} \right) = 0$.

25. Let (x_n) be a bounded sequence and let $s = \sup\{x_n : n \in \mathbb{N}\}$. If $s \notin \{x_n : n \in \mathbb{N}\}$, then there subsequence of (x_n) that converges to s.

26. Define Cantor set and show that it contains no nonempty open interval as a subset.

27. Write short notes on *Steriographic projection* and *Riemann sphere.*

28. Prove the triangle inequality $|z_1 - z_2| \leq |z_1| + |z_2|$ for complex numbers.

(5 × 2 = 10 Weighta

Essay Questions, Answer any **two** *questions from three, weightage 4 each.*

29. (a) Let (x_n) be a sequence of positive real numbers such that $L := \lim(x_{n+1} / x_n)$ exists. If L show that $\lim(x_n) = 0$.

(b) Find $\lim_{n \to \infty} \left(\frac{2^{3n}}{3^{2n}} \right)$.

30. State and prove monotone subsequence theorem and deduce the Bolzano-Weierstrass Theore

31. Show that a subset of $\mathbb{R}$ is open if and only if it is the union of countably many disjoint o intervals in $\mathbb{R}$.

(2 × 4 = 8 Weight