D 50602 (Pages : 3) Name.........................

Reg. No.........................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(CUCBCSS–UG)

MAT 5B 08—DIFFERENTIAL EQUATIONS

Time : Three Hours Maximum : 120 Marks

Section A

Answer **all the twelve** *questions.*
Each question carries 1 mark.

1. L (t) = ————.

2. Show that $y = e^{-t}$ is a solution of the differential equation $\frac{d^2y}{dt^2} - y = 0$.

3. Write the 2-dimensional wave equation.

4. Show that the equation $\frac{ydx - xdy}{y^2}$ is exact.

5. Find the order of the differential equation : $\frac{t\,d^2y}{dx^2} + 2y = \cos 3t$.

6. Find $W\left(t, t^{-1}\right)$.

7. Write the characteristic equation of the differential equation $\frac{d^2y}{dt^2} - \frac{4\,dy}{dt} + 5y = 0$.

8. Find $L^{-1}\left[\frac{1}{s^2 + a^2}\right]$.

9. Find the general solution of the differential equation $\frac{d^2x}{dt^2} + 16x = 0$.

10. When we say that a function is odd ?

11. Solve : $\frac{dy}{dx} + y^2 = 0$

12. Is $\begin{bmatrix} 1 \\ \alpha \end{bmatrix}$ is an eigen vector of the matrix $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$. Justify your answer.

(12 × 1 = 12 marks)

Turn over

Section B

Answer any **ten** *out of fourteen questions.*
Each question carries 4 marks.

13. Solve : $\frac{d^2y}{dt^2} - \frac{6dy}{dt} + 9y = 0$.

14. Find $L^{-1}\left[\frac{1}{s^2(s^2+4)}\right]$.

15. Find the Fourier coefficients corresponding to the function $f(t) = 1, 0 \le t \le 2\pi$.

16. Find the interval in which the initial value problem $ty^1 + 2y = 4t^2, y(1) = 2$ has a unique solution.

17. Solve : $(y\cos x + 2x e^y) + (\sin x + x^2 e^y - 1)y^1 = 0$.

18. Given that Y_1 and Y_2 are solutions of the homogeneous equation $y'' + p(t)y' + q(t)y = 0$. Show that $2y_1 + 3y_2$ is also a solution of $y'' + p(t)y' + q(t)y = 0$.

19. Show that $W(e^{\lambda t}\cos\mu t, e^{\lambda t}\sin\mu t) = \mu e^{2\lambda t}$.

20. Find a particular solution of $y'' - 3y' - 4y = 3e^{2t}$.

21. Let n be a +ve integer ; show that $L(t^n) = \frac{n!}{s^{n+1}}, s > 0$.

22. Find the Fourier sine series for the function $f(t) = 1, 0 \le t \le \pi$.

23. What is "Linearization" ? Give an example.

24. Find the value of b for which the equation $(xy^2 + bx^2y)dx + (x+y)x^2dy = 0$ is exact and then solve using that value of b.

25. Find a differential equation whose general solution is $y = c_1e^{2t} + c_2e^{-2t}$.

26. Find the inverse transform of $H(s) = \frac{a}{s^2(s^2+a^2)}$ by convolution integral.

(10 × 4 = 40 marks)

Section C

*Answer any **six** out of nine questions.*
Each question carries 7 marks.

27. Find the general solution of $y'''' + y''' - 7y'' - y' + 6y = 0$. Also find the solution that satisfies the initial conditions $y(0) = 1, y'(0) = 0, y''(0) = -2$, and $y'''(0) = -1$.

28. Given that $y_1 = e^t, y_2 = te^t, y_3 = e^{-t}$ are solutions of the homogeneous equation corresponding to $y''' - y'' - y' + y = g(t)$, determine a particular solution in terms of an integral.

29. Using the method of undetermined coefficients, solve : $y'' + 4y = 3\sin 2t$.

30. Find the fundamental set of solutions for the differential equation $y'' - y = 0$ using the initial point $t_0 = 0$.

31. State and prove "Abel's theorem".

32. State and prove "The Convolution Integral Theorem.

33. Using Laplace transforms, solve the initial value problem $\frac{d^2y}{dt^2} + \frac{2\,dy}{dt} - 3y = \sin t$ given that $y(0) = y'(0) = 0$.

34. Evaluate : $\int_0^\infty \frac{e^{-t} - e^{-2t}}{t}\, dt$.

35. Suppose that a mass weighing 10 *lb* stretches a spring 2 in. If the mass is displaced an additional 2 in. and is then set in motion with an initial upward velocity of 1 ft/sec. Determine the position of the mass at any later time. Also, determine the period, amplitude of the motion.

(6 × 7 = 42 marks)

Section D

*Answer any **two** out of three questions.*
Each question carries 13 marks.

36. Solve : $y'' + 4y = 3\csc t$.

37. The motion of a spring-mass system is governed by the differential equation $u'' + 0.125\,u' + u = 0$, where u is measured in feet and t in seconds. If $u(0) = 2$ and $u'(0) = 0$, determine the position of the mass at any time. Also find the quasi frequency and the quasi period time at which the mass first passes through the equilibrium position.

38. Find the Fourier series to represent $f(x)$ in $[-\pi, \pi]$, given $f(x) = x + x^2$ in $-\pi < x < \pi$; and $f(x) = \pi^2$ when $x = \pm\pi$. Also deduce that $1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots = \frac{\pi^2}{6}$.

(2 × 13 = 26 marks)